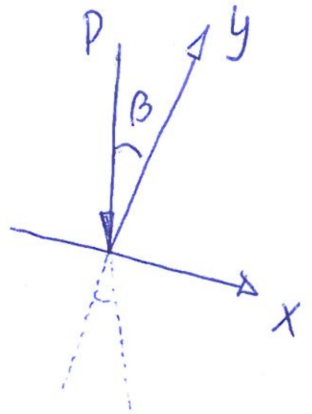
2.1




$P = 800\,N$

$\tan\beta = \frac{3}{4}$

$P_x = P \cdot \sin\beta$

$P_y = -P \cdot \cos\beta$

$\beta = 36{,}86...^\circ$

$P_x = 800 \cdot \sin 36{,}86 = 480\,N$

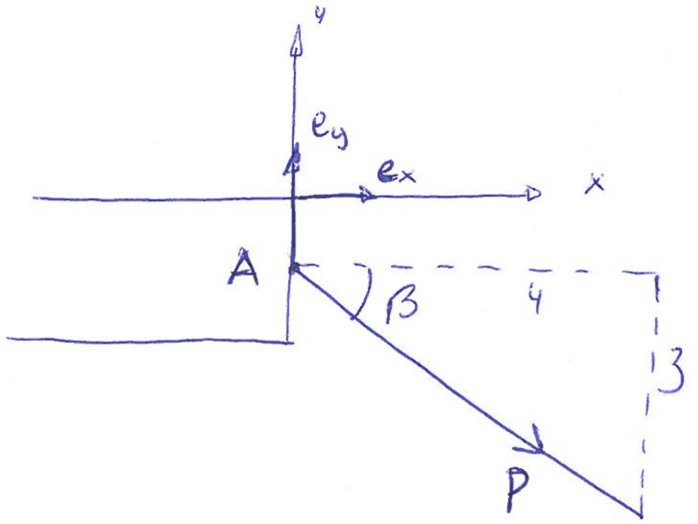
$P_y = -800 \cdot \cos\beta = -640\,N$

Svar: $P_x = 480N$, $P_y = -640\,N$

$\bar{P} = (480\,\hat{e}_x - 640\,\hat{e}_y)N$

---

2.2




$P = 10\,kN$

Basvektorer $\bar{e}_x$ $\bar{e}_y$

$\tan\beta = \frac{3}{4}$

$\beta = 36{,}86...^\circ$

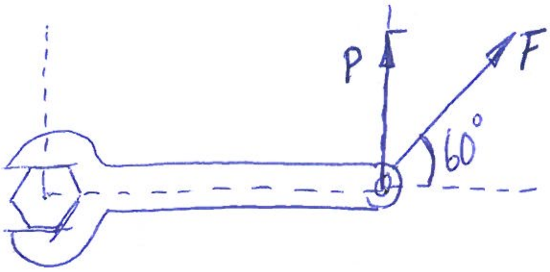
$\sin\beta = \frac{3}{5}$

$\cos\beta = \frac{4}{5}$

$\bar{P} = \left(P \cdot \cos\beta \cdot \bar{e}_x - P\sin\beta \cdot \bar{e}_y\right) = \left(10 \cdot \cos\beta\,\bar{e}_x - 10\sin\beta\,e_y\right)$

$\bar{P} = \left(8\,\bar{e}_x - 6\,\bar{e}_y\right) kN$

2,3




$F_x = ?$ $\quad F = ?$

$F_y = P$

$F_x = F \cdot \cos 60$

$F_x = \dfrac{F}{2}$

$F_y = F \cdot \sin 60°$

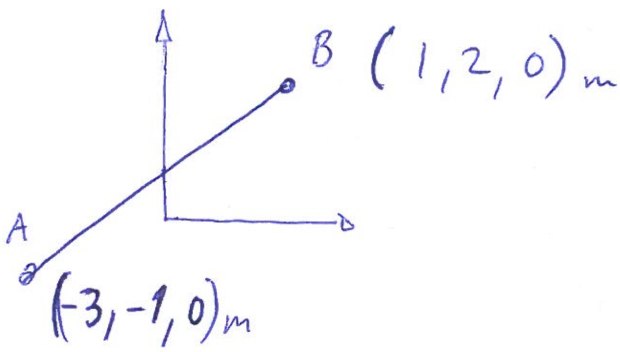
$P = \dfrac{\sqrt{3}}{2} F \Rightarrow F = \dfrac{2P}{\sqrt{3}}$

$F_x = \dfrac{2P}{\sqrt{3}} \cdot \dfrac{1}{2} = \dfrac{P}{\sqrt{3}}$

Svar: $F_x = \dfrac{P}{\sqrt{3}}$ $\qquad F = \dfrac{2P}{\sqrt{3}}$

---

2,4



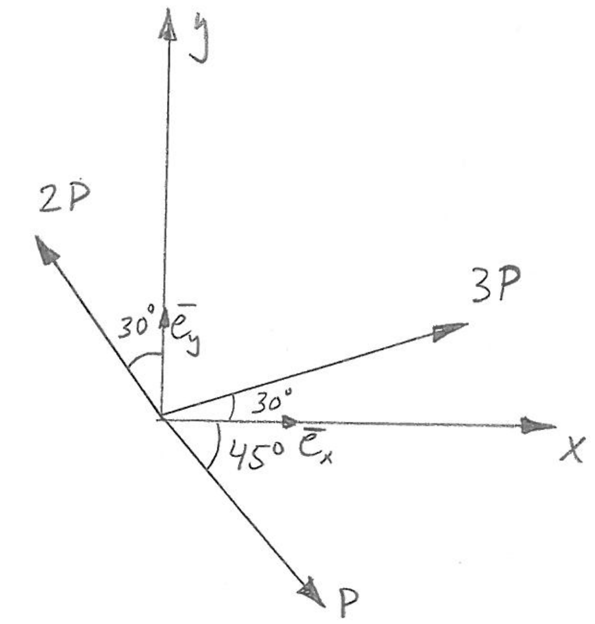

$\bar{e}_{AB} = \cancel{\dfrac{\bar{r}_B - \bar{r}_A}{\sqrt{\ }}} \; \dfrac{\bar{r}_B - \bar{r}_A}{|\bar{r}_B - \bar{r}_A|} \; \dfrac{4,\ 3,\ 0}{\sqrt{4^2+3^2}} = \dfrac{(4,3,0)}{5}$

$\bar{e}_{AB} = \dfrac{1}{5}(4,3,0)$

$S_A = 50 \cdot \left(\dfrac{4}{5}\bar{e}_x + \dfrac{3}{5}\bar{e}_y\right) = (40\bar{e}_x + 30\bar{e}_y)$

$S_B = (-40\bar{e}_y + 30\bar{e}_y)$ N

2.5




$$\bar{F}_1 = P\cos 45°\,\bar{e}_x - P\sin 45°\,\bar{e}_y$$
$$\bar{F}_2 = 3P\cos 30°\,\bar{e}_x + 3P\sin 30°\,\bar{e}_y$$
$$\bar{F}_3 = -2P\sin 30°\,\bar{e}_x + 2P\cos 30°\,\bar{e}_y$$

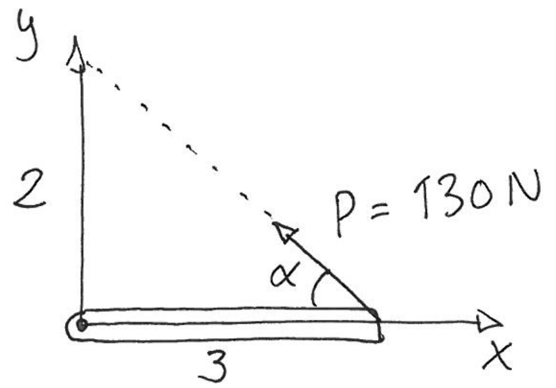
Kan också skrivas som.

$$\bar{F}_1 = P\cdot\frac{\sqrt{2}}{2}\,\bar{e}_x - P\cdot\frac{\sqrt{2}}{2}\,\bar{e}_y$$
$$\bar{F}_2 = 3P\cdot\frac{\sqrt{3}}{2}\,\bar{e}_x + 3P\,\frac{1}{2}\,\bar{e}_y$$
$$\bar{F}_3 = -2P\,\frac{1}{2}\,\bar{e}_x + 2\,\frac{\sqrt{3}}{2}\,\bar{e}_y$$

eller
$$\bar{F}_1 = \frac{\sqrt{2}P}{2}\left(\bar{e}_x - \bar{e}_y\right)$$
$$\bar{F}_2 = \frac{3}{2}P\left(\sqrt{3}\,\bar{e}_x + \bar{e}_y\right)$$
$$F_3 = P\left(-\bar{e}_x + \sqrt{3}\,\bar{e}_y\right)$$

$$\bar{F} = \bar{F}_1 + \bar{F}_2 + \bar{F}_3 = \left(\frac{\sqrt{2}P}{2} + \frac{3\sqrt{3}P}{2} - P\right)\bar{e}_x + \left(-\frac{\sqrt{2}P}{2} + \frac{3P}{2} + \sqrt{3}P\right)\bar{e}_y$$

2.6)




$\tan\alpha = \frac{2}{3}$

$\sin\alpha = \frac{2}{\sqrt{13}}$

$\cos\alpha = \frac{3}{\sqrt{13}}$

$\alpha = 33{,}69°$

$X: \rightarrow : -130 \cdot \cos 33{,}69 = -108{,}16\ N$

$y: \uparrow : 130 \cdot \sin 33{,}69 = 72{,}1\ N$

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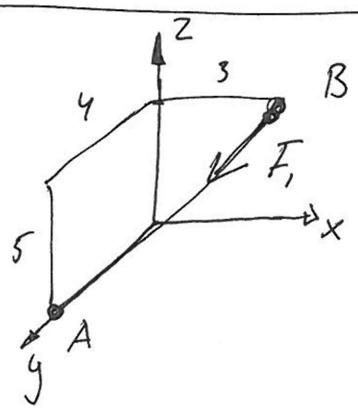
2.10)




$F_1 = 300\ N$

$\bar{F}_1 = F_1 \cdot \hat{e}_{BA}$

$$\hat{e}_{BA} = \frac{(4,0,0)-(0,3,5)}{|\bar{r}_A - \bar{r}_B|}$$

$$\hat{e}_{BA} = \frac{(4,-3,-5)}{\sqrt{4^2 + -3^2 + (-5)^2}} = \frac{(4,-3,-5)}{\sqrt{50}}$$

$$\hat{e}_{BA} = \frac{(4,-3,-5)}{\sqrt{25}\cdot\sqrt{2}} = \frac{(4,-3,-5)}{5\cdot\sqrt{2}}$$

$$\bar{F}_1 = \frac{300}{5\cdot\sqrt{2}}(4,-3,-5)\ N$$

$$\bar{F}_1 = \frac{60}{\sqrt{2}}(4,-3,-5)\ N$$

Repets kraft på mannen är $-\bar{F}_1$

2.11 $\bar{e}_{AB} = \dfrac{(0,5,2)-(3,4,0)}{|\quad|} = \dfrac{-3,1,2}{\sqrt{14}}$

$$\bar{e}_{AC} = \frac{(4,0,5)-(3,4,0)}{|\quad|} = \frac{(1,-4,5)}{\sqrt{42}}$$

$$\bar{e}_{AB} \circ \bar{e}_{AC} = \cos\beta$$

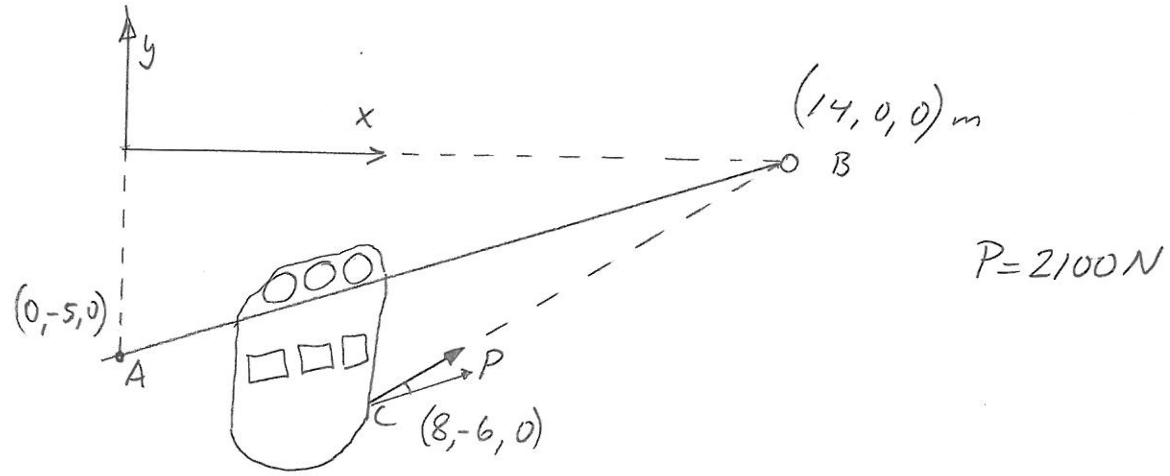
$$\cos\beta = \frac{1}{\sqrt{42}} \cdot \frac{-3}{\sqrt{14}} + \frac{1(-4)}{\sqrt{42}\cdot\sqrt{14}} + \frac{10}{\sqrt{42}\cdot\sqrt{14}}$$

$$\cos\beta = \frac{-3}{\sqrt{14}\cdot\sqrt{3}\cdot\sqrt{14}} + \frac{4}{\sqrt{3}\cdot\sqrt{14}\cdot\sqrt{14}} + \frac{10}{\sqrt{14}\cdot\sqrt{14}\cdot\sqrt{3}}$$

$$\cos\beta = \frac{3}{14\cdot\sqrt{3}}$$

---

2,12




$$\bar{e}_{CB} = \frac{(14,0,0)-(8,-6,0)}{|\quad|} = \frac{(6,6,0)}{\sqrt{6^2+6^2}} = \frac{(6,6,0)}{\sqrt{72}}$$

$$\bar{e}_{AB} = \frac{(14,0,0)-(0,-5,0)}{|\quad|} = \frac{(14,5,0)}{\sqrt{14^2+5^2}} = \frac{(14,5,0)}{\sqrt{221}}$$

$$\bar{e}_{CB} = \frac{(6,6,0)}{\sqrt{72}} = \frac{(6,6,0)}{\sqrt{36}\cdot\sqrt{2}} = \frac{6}{6\cdot\sqrt{2}}(1,1,0) = \frac{1}{\sqrt{2}}\cdot(1,1,0)$$

$$\bar{P} = P\cdot\bar{e}_{CB} = \frac{2100}{\sqrt{2}}(1,1,0)$$

Krafters komponent avseende på linan AB

$$P_{Komp} = \bar{P}\cdot\bar{e}_{AB} = P\cdot\bar{e}_{CB}\cdot\bar{e}_{AB} = \frac{2100}{\sqrt{2}\cdot\sqrt{221}}(14+5+0)\,N$$

$$= \frac{19}{\sqrt{442}}\;2100\,N$$

2.13

$F_1 = 340\,N$

$F_2 = 420\,N$

$F_3 = 390\,N$

Ta reda på kraftvektorerna $\bar{F}_1$ $\bar{F}_2$ och $\bar{F}_3$

Först riktningsvektorerna. $\hat{e}_{F_1}$ $\hat{e}_{F_2}$ $\hat{e}_{F_3}$

$\hat{e}_{F_1}$ $\hat{e}_{F_2}$ $\hat{e}_{F_3}$

$$\hat{e}_{F_1} = \frac{(9,-8,0)-(0,0,12)}{|(9,-8,0)-(0,0,12)|} = \frac{(9,-8,12)}{\sqrt{81+64+144}} = \frac{(9,-8,12)}{17}$$

$$\Rightarrow \bar{F}_{F_1} = \hat{e}_{F_1} \cdot F_1 = \frac{(9,-8,12)}{17} \cdot 340 = 20(9,-8,-12)\,N$$

$$\hat{e}_{F_2} = \frac{(4,6,0)-(0,0,12)}{\sqrt{16+36+144}} = \frac{(4,6,-12)}{14}$$

$$\hat{e}_{F_3} = \frac{(-5,0,0)-(0,0,12)}{\sqrt{25+144}} = \frac{(-5,0,-12)}{13}$$

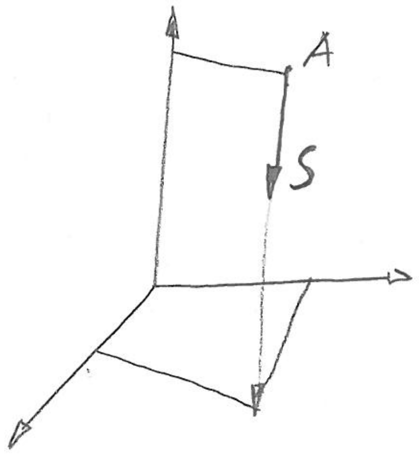
$$\bar{F}_{F_2} = \hat{e}_{F_2} \cdot F_2 = \frac{(4,6,-12)}{14} \cdot 420 = 30(4,6,-12)\,N$$

$$\bar{F}_{F_3} = \hat{e}_{F_3} \cdot F_3 = \frac{(-5,0,-12)}{13} \cdot 390 = (150,0,-360)$$

$$\bar{R} = (\Sigma F_x, \Sigma F_y, \Sigma F_z) = (180+120-150, -160+180, -240-360-360)$$

$$\bar{R} = (150, 20, -960)\,N$$

2.14)




$\bar{S} = ?$

$S = 0{,}9\,kN$

$\bar{r}_A = (0, 2, 7)$

$\bar{r}_B = (4, 6, 0)$

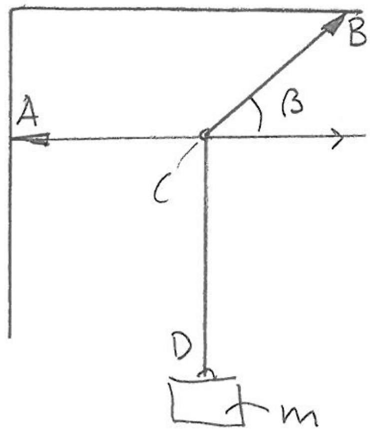
$$\bar{S} = S \cdot \hat{e}_{AB}$$

$$\hat{e}_{AB} = \frac{\bar{r}_B - \bar{r}_A}{|\bar{r}_B - \bar{r}_A|} = \frac{(4,6,0)-(0,2,7)}{|(4,6,0)-(0,2,7)|}$$

$$= \frac{(4,4,-7)}{\sqrt{4^2+4^2+7^2}} = \frac{1}{9}(4,4,-7)$$

$$\bar{S} = \frac{0{,}9}{9} \cdot (4, 4, -7)\,kN = (0{,}4,\ 0{,}4,\ -0{,}7)\,kN$$

2.17




$S_{CB}$

$S_{CA}$

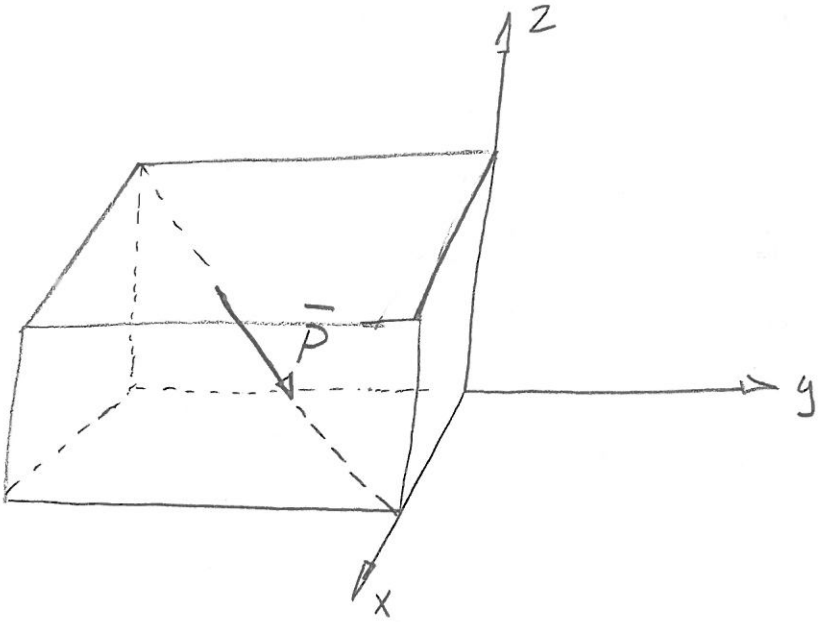
$\rightarrow: S_{CB}\cos\beta - S_{CA} = 0$

$\uparrow: S_{CB}\sin\beta - mg = 0$

$$S_{CB} = \frac{mg}{\sin\beta}$$

$$S_{CA} = \frac{mg\cos\beta}{\sin\beta} = \frac{mg}{\tan\beta}$$

2,19)




$P = 200N$

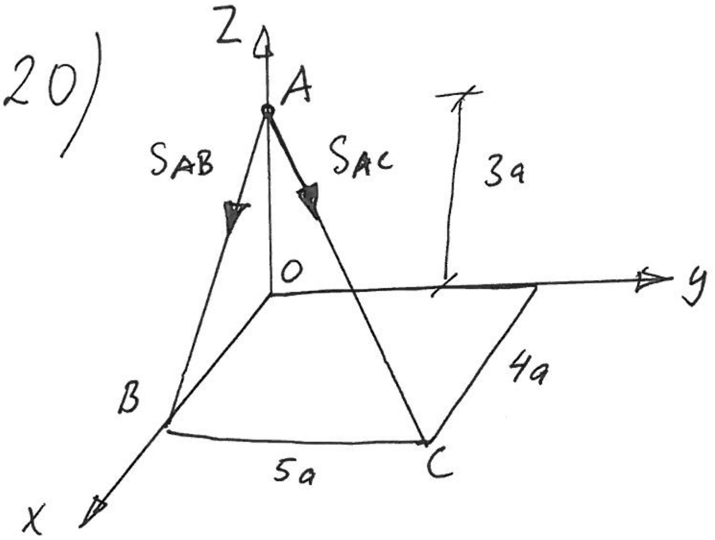
$\bar{r}_{AB} = (2,0,0) - (0,-4,3) = (2,4,-3)$

$\hat{e}_{AB} = \dfrac{(2,4,-3)}{\sqrt{\underset{4}{2^2}+\underset{16}{4^2}+3^2}} = = \dfrac{1}{\sqrt{29}}(2,4,-3)$

$\bar{P} = P \cdot \hat{e}_{AB} = \dfrac{200}{\sqrt{29}}(2,4,-3)\,N$

$\bar{P} = \dfrac{200}{\sqrt{29}}\left(2\hat{e}_x + 4\hat{e}_y - 3\hat{e}_z\right)N$

2.20)




Sökt total kraften vid $A = R$.

$S_{AB} = P$

$S_{AC} = \sqrt{2}P$

$$\bar{S}_{AB} = S_{AB} \cdot \hat{e}_{AB}$$

$$\bar{S}_{AC} = S_{AC} \cdot \hat{e}_{AC}$$

$$\hat{e}_{AB} = \frac{(\bar{r}_B - \bar{r}_A)}{|\bar{r}_B - \bar{r}_A|} = \frac{(4a,0,0)-(0,0,3a)}{|(4a,0,0)-(0,0,3a)|} = \frac{(4a,0,-3a)}{\sqrt{(4a)^2+(3a)^2}}$$

$$\hat{e}_{AB} = \frac{(4a,0,-3a)}{\sqrt{16a^2+9a^2}} = \frac{(4a,0,-3a)}{5a} = \left(\frac{4}{5}, 0, -\frac{3}{5}\right)$$

$$\hat{e}_{AC} = \frac{\bar{r}_c - \bar{r}_A}{|\bar{r}_c - \bar{r}_A|} = \frac{(4a,5a,0)-(0,0,3a)}{|(4a,5a,0)-(0,0,3a)|} = \frac{(4a,5a,-3a)}{\sqrt{16a^2+25a^2+9a^2}}$$

$$\hat{e}_{AC} = \left(\frac{4}{5\cdot\sqrt{2}}, \frac{1}{\sqrt{2}}, -\frac{3}{5\sqrt{2}}\right)$$

$$\bar{S}_{AB} = P\cdot\left(\frac{4}{5}, 0, -\frac{3}{5}\right) = \left(\frac{4P}{5}, 0, -\frac{3P}{5}\right)$$

$$\bar{S}_{AC} = \sqrt{2}P\left(\frac{4}{5\cdot\sqrt{2}}, \frac{1}{\sqrt{2}}, -\frac{3}{5\cdot\sqrt{2}}\right) = \left(\frac{4P}{5}, P, -\frac{3P}{5}\right)$$

$$\bar{R} = \bar{S}_{AB} + \bar{S}_{AC} = \left(\frac{4P}{5}, 0, -\frac{3P}{5}\right) + \left(\frac{4P}{5}, P, -\frac{3P}{5}\right)$$

$$\bar{R} = \left(\frac{8P}{5}, P, -\frac{6P}{5}\right) \qquad R = \sqrt{\left(\frac{8P}{5}\right)^2 + P^2 + \left(\frac{6P}{5}\right)^2}$$

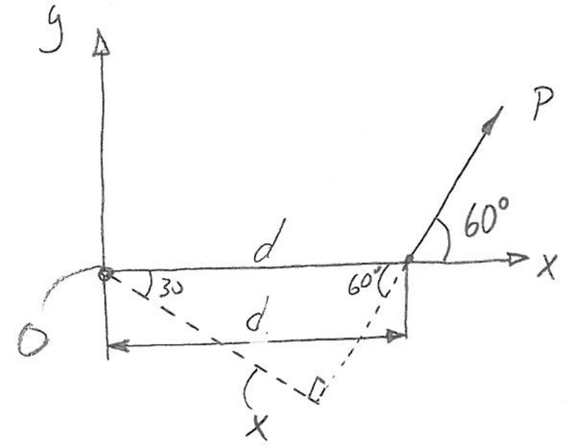
2,20 forts.

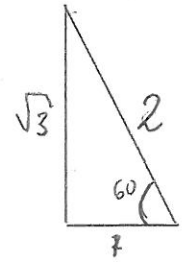
$$R = \sqrt{\frac{64P^2}{25} + P^2 + \frac{36P^2}{25}} = \sqrt{\frac{64P^2 + 25P^2 + 36P^2}{25}}$$

$$R = \sqrt{\frac{125P^2}{25}} = \underline{\underline{P \cdot \sqrt{5}}}$$

Svar: Resultanten är $P \cdot \sqrt{5}$

2,26) a)

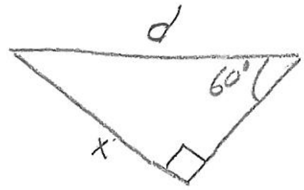



$\sin = \frac{\sqrt{3}}{2}$

$\cos 60 = \frac{1}{2}$

hävarmen till kraften P map O.




$= \quad \sin 60 = \frac{x}{d}$

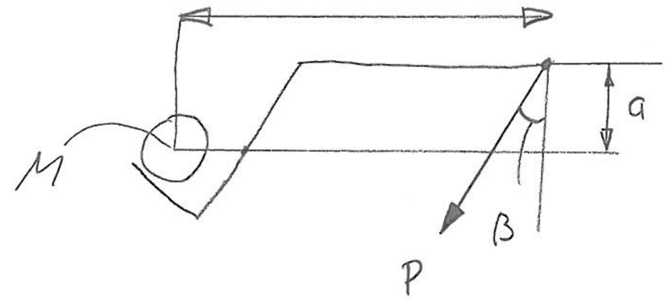
$\Rightarrow x = d \sin 60°$

$x = \frac{d \cdot \sqrt{3}}{2}$

$$\overset{\curvearrowleft}{M}_o = P \cdot \frac{d \cdot \sqrt{3}}{2}$$

$$\overset{\curvearrowleft}{M}_o = \frac{\sqrt{3}\, P \cdot d}{2}\, \hat{e}_z$$

2.29




Hur stort moment Bidrar P med i $M_A$

$$\overset{\frown}{M_A} = P \cdot \cos\beta \cdot b - P \cdot \sin\beta \cdot a$$

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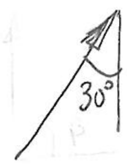
2,30] P kraftens vridande förmåga

är y komposanten riktad uppåt




$\beta = 30$


$$\sin\beta = \frac{P_y}{P} \Rightarrow P_y = P \cdot \cos\beta = \frac{\sqrt{3}P}{2}$$

$$\overset{\frown}{M_o} = \frac{\sqrt{3}P}{2} \cdot r = \frac{\sqrt{3} \cdot 60}{20} \cdot 0{,}08 = \underline{\underline{4{,}16\ Nm}}$$

2.32 P:s vridande förmåga map 0.


$$\overset{\curvearrowright}{M_0} = P \cdot \cos\beta \cdot d \cos\alpha - P \cdot \sin\beta \cdot d \sin\alpha$$

$\alpha = 30^\circ$

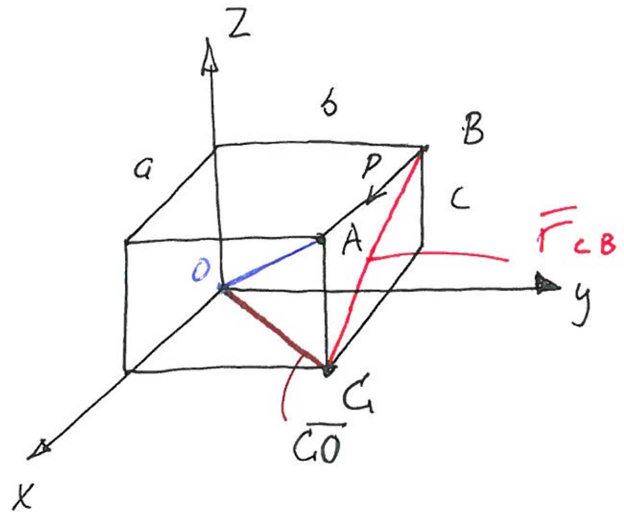
$\beta = 30^\circ$

$$\overset{\curvearrowright}{M_0} = \frac{\cancel{2} \cdot \sqrt{3}}{\cancel{2}} \cdot \frac{0{,}025 \cdot \sqrt{3}}{2} - \frac{\cancel{2} \cdot 1}{\cancel{2}} \cdot \frac{0{,}025 \cdot 1}{2}$$

$$\overset{\curvearrowright}{M_0} = \frac{3 \cdot 0{,}025}{2} - \frac{0{,}025 \cdot 1}{2} = \frac{\cancel{2} \cdot 0{,}025}{\cancel{2}} \text{ Nm}$$

$$\overset{\curvearrowright}{M_0} = 0{,}025 \text{ Nm}$$

2.37




$$\bar{P} = P \cdot \hat{e}_{BA}$$

$$\hat{e}_{BA} = \frac{\bar{r}_{BA}}{|\bar{r}_{BA}|} = \frac{(a,b,c)-(0,b,c)}{\sqrt{a^2}} = \frac{a(1,0,0)}{a} = (1,0,0)$$

$$\bar{P} = P \cdot (1,0,0) = (P,0,0)$$

$$\bar{M}_o = \bar{r} \times \bar{P} = (a,b,c) \times (P,0,0)$$

$$= \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ a & b & c \\ P & 0 & 0 \end{vmatrix} = (0, c \cdot P, -b \cdot P) = c \cdot P \hat{e}_y - b \cdot P \hat{e}_z$$

$\bar{M}_B = \bar{0}$ för $\bar{P}$ vektorn har ingen "momentarm" i punkten B.

$$\bar{M}_c = \bar{M}_o + \overline{CO} \times \bar{P}$$ $\boxed{\text{jämför ekv 2.15 sid 37}}$

$$\overline{CO} = (-a, -b, 0)$$ (spets – fotpunkt)
origo punkt C

$$\overline{CO} \times \bar{P} = \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ -a & -b & 0 \\ P & 0 & 0 \end{vmatrix} = (0, 0, b \cdot P)$$

$\Rightarrow$ forts.

2.37 forts

$$\bar{M}_c = (0,\ c\cdot P,\ -b\cdot P) + (0,0,b\cdot P)$$

$$\bar{M}_c = (0,\ cP,\ 0)$$

Alternativ lösning:

$$\bar{M}_C = \bar{r}_{CB} \times \bar{P}$$

jämför ekvation 2,9 sidan 23

A = den punkt vi vill räkna moment.

$$\bar{M}_A = \bar{r}_{A\underline{P}} \times \bar{F}$$

kraftens angrepps punkt.

$$\bar{r}_{CB} = (0,b,c) - (a,b,0) = (-a,0,c)$$

$$\bar{M}_c = (-a,0,c) \times (P,0,0) = \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ -a & 0 & c \\ P & 0 & 0 \end{vmatrix}$$

$$= (0\cdot 0 - c\cdot 0,\ -(-a\cdot 0 - c\cdot P),\ -a\cdot 0 - 0\cdot P) = (0,\ cP,\ 0)$$

Momentet kring punkten C kan alltså lösas på två sätt

Alt lösning igen:

2, 37 $\qquad \overline{M}_c = \overline{r}_c \times \overline{P}$

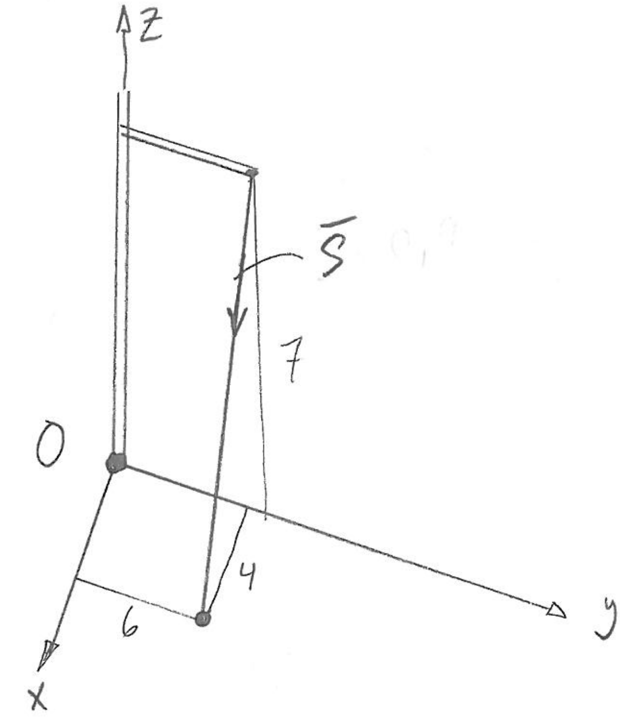
$$\overline{P} = (P, 0, 0)$$

$$\overline{r}_{CA} = (a, b, c) - (a, b, 0) = (0, 0, c)$$

$$\overline{M}_c = \overline{r}_{CA} \times \overline{P} = \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ 0 & 0 & c \\ P & 0 & 0 \end{vmatrix} = (0, c \cdot P, 0)$$

$$\overline{M}_c =$$

2,39




$S = 0{,}9\,kN$

$$\boxed{\bar{M}_o = \bar{r} \times \bar{F}}$$

$M_o$ söks

$$\bar{F} = \bar{S} = S \cdot \hat{e}_{AB} \Rightarrow$$

$$\hat{e}_{AB} = \frac{(4,6,0)-(0,2,7)}{|\qquad\qquad|} = \frac{4,4,-7}{\sqrt{16+16+49}} = \frac{1}{9}(4,4,-7)$$

$$\hat{e}_{AB} = \frac{(4,4,-7)}{9} \Rightarrow \bar{F} = \frac{900}{9}(4,4,-7) = (400,400,-700)\,N$$

$$\bar{r}_A = (0,2,7)\,m$$

$$\bar{M}_o = \bar{r} \times \bar{F} = (0,2,7) \times (400,400,-700)\,Nm$$

$$= 100 \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ 0 & 2 & 7 \\ 4 & 4 & -7 \end{vmatrix} Nm = 100(-42\hat{e}_x, 28\hat{e}_y, -8\hat{e}_z)\,Nm$$

$$= 200(-21\hat{e}_x, 14\hat{e}_y, -4\hat{e}_z)\,Nm$$

2.48)

$\tan\beta = \frac{3}{4}$

$\sin\beta = \frac{3}{5}$

$\cos\beta = \frac{4}{5}$

$\rightarrow x: \quad 250 \cdot \frac{3}{5} - 100 = 50\,N$

$\uparrow y: \quad 250 \cdot \frac{4}{5} - 100 = 100$

$$\bar{F} = (50, 100, 0)\,N$$

$$M_0 = (0, 0, 60)\,Nm$$

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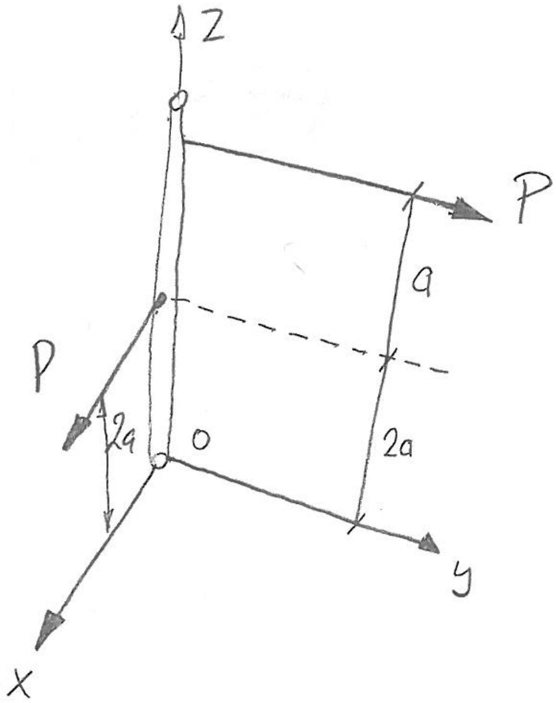
2.52)




$x: \swarrow = P - 0 = P$

$y: \searrow = P - 0 = P$

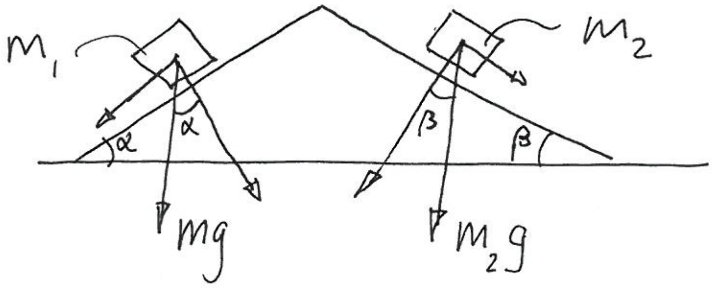
$M_x = -P \cdot 3 \cdot a = -P \cdot 3a\,\hat{e}_x$

$M_y = 2a \cdot P = P \cdot 2a\,\hat{e}_y$

$\bar{F} = P\hat{e}_x + P\hat{e}_y$

$\bar{M} = -3Pa\hat{e}_x + 2Pa\hat{e}_y$

3,1]




$m_1 = m$ är givet sökt $m_2$

Lådorna i jämvikt ger:

$$mg \cdot \sin\alpha = m_2 \cdot g \cdot \sin\beta$$

$$m_2 = \frac{m \cdot \sin\alpha}{\sin\beta}$$

---

3,2]

$\curvearrowleft O$ :

$$S \cdot \cos\alpha \cdot L \sin\beta + \overbrace{\underbrace{S \sin\alpha \cdot L \cos\beta}}^{S \cdot \cos\beta} - mg \cdot \underbrace{\frac{L}{2} \cdot \sin\beta}_{hävarm} = 0$$

$$S = \frac{mg \cdot \sin\beta}{2\left(\cos\alpha \cdot \sin\beta + \sin\alpha \cdot \cos\beta\right)}$$

2

$\beta = \frac{\pi}{4}$

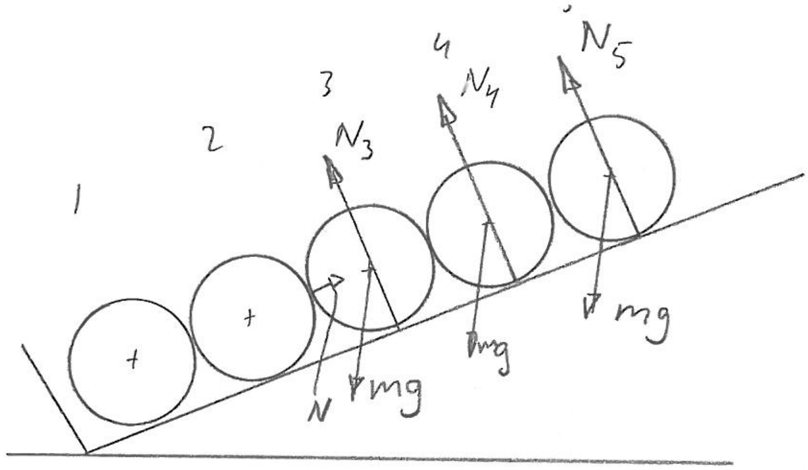
$\tan\alpha = \frac{4}{3}$

$$S = \frac{mg \cdot \frac{\sqrt{2}}{2}}{\cancel{2}\left(\frac{3}{5} \cdot \frac{\sqrt{2}}{\cancel{2}} + \frac{4}{5} \cdot \frac{\sqrt{2}}{\cancel{2}}\right)} = \frac{mg \cdot \frac{\sqrt{2}}{2}}{\frac{3 \cdot \sqrt{2}}{5} + \frac{4 \cdot \sqrt{2}}{5}}$$

$$= \frac{mg \cdot \frac{\sqrt{2}}{2} \cdot 5}{3 \cdot \sqrt{2} + 4\sqrt{2}} = \frac{mg \cdot 5}{\frac{2}{\cancel{\sqrt{2}}}\left(3 \cdot \cancel{\sqrt{2}} + \cancel{\sqrt{2}} \cdot 4\right)}$$

$$= \frac{mg \cdot 5}{6 + 8} = \frac{5}{14} mg$$

3,7]



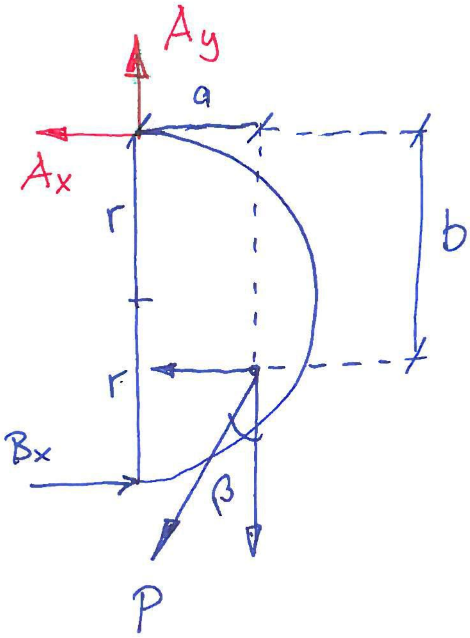

Jämvikt för det frilagda systemet

$\nearrow$ : $N - 3mg \sin\beta = 0$

$\Rightarrow N = 3mg \sin\beta$

3,8]

Sökt $A_x$ o $A_y$




$$\boxed{\begin{matrix}\Sigma \bar{F} = 0 \\ \Sigma \bar{M} = 0\end{matrix}} \Rightarrow$$

$\rightarrow$: $B_x - P\sin\beta - A_x = 0$ (I)

$\uparrow$: $A_y - P\cdot\cos\beta = 0$

$\Rightarrow A_y = P\cdot\cos\beta$

$\curvearrowleft A$: $B_x\cdot 2r - P\cdot\sin\beta\cdot b - P\cdot\cos\beta\cdot a = 0$

$$B_x = \frac{P\cdot(b\cdot\sin\beta + a\cos\beta)}{2r}$$ (III) ins i (I)

$$\frac{P\cdot b\cdot\sin\beta}{2r} + \frac{P\cdot a\cdot\cos\beta}{2r} - P\cdot\sin\beta - A_x = 0$$

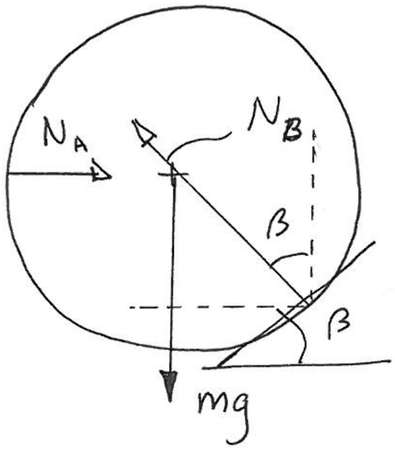
$$A_x = \frac{P}{2r}\left(b\cdot\sin\beta + a\cdot\cos\beta - 2r\cdot\sin\beta\right)$$

$$A_x = \frac{P}{2r}\left((b-2r)\sin\beta + a\cos\beta\right)$$

Svar: $A_x = \frac{P}{2r}\left((b-2r)\sin\beta + a\cos\beta\right)$

$A_y = P\cos\beta$

3.10 Lösning frilägg sätt ut krafter




$\rightarrow$: $N_A - N_B \sin\beta = 0$ I

$\uparrow$: $N_B \cdot \cos\beta - mg = 0$ II

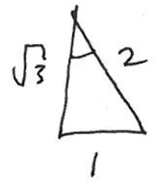
$N_B = \dfrac{mg}{\cos\beta}$ ins i (I) $\Rightarrow$

$N_A = \dfrac{mg}{\cos\beta} \cdot \sin\beta = mg \tan\beta$

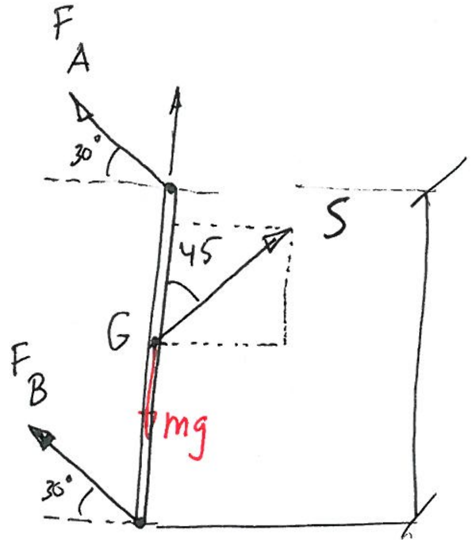
<u>Svar:</u> $N_A = mg \tan\beta$

$N_B = \dfrac{mg}{\cos\beta}$

3,12




$\cos 30 = \frac{\sqrt{3}}{2}$




$\rightarrow \quad S \cdot \frac{\sqrt{2}}{2} - F_A \cdot \cos 30 - F_B \cdot \cos 30 = 0 \quad I$

$\uparrow : \quad F_A \cdot \sin 30° + F_B \cdot \sin 30° + \frac{S \cdot \sqrt{2}}{2} - mg = 0 \quad II$

$\left( \curvearrowright B : \quad \frac{S \cdot \sqrt{2}}{2} \cdot \frac{d}{2} - d \cdot F_A \cdot \cos 30° = 0 \right)$

$\curvearrowleft G : \quad \frac{d}{2} \cdot \frac{\sqrt{3}}{2} F_A - \frac{d}{2} \frac{\sqrt{3}}{2} F_B = 0 \quad III$

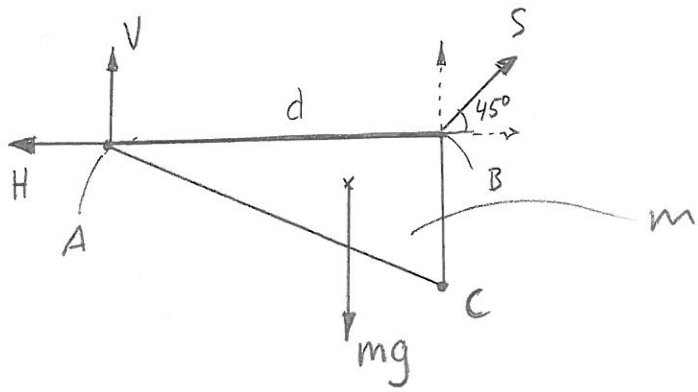
$F_A = F_B \Rightarrow$ i $I$

$\frac{S \cdot \sqrt{2}}{2} - F_A \cdot \frac{\sqrt{3}}{2} - F_A \cdot \frac{\sqrt{3}}{2} = 0 \Rightarrow F_A = \frac{\sqrt{2}}{2 \cdot \sqrt{3}} S$

ins i $II$ $\quad \frac{\sqrt{2}}{2 \cdot \sqrt{3}} S + \frac{\sqrt{2}}{2} S - mg = 0$

$S = \frac{mg \cdot 2 \cdot \sqrt{3}}{\sqrt{2} + \sqrt{6}}$

3,13]




$\rightarrow$ : $S \cdot \cos 45 - H = 0$

$\uparrow$ : $S \cdot \sin 45 + V - mg = 0$

$\overset{\curvearrowright}{B}$ : $V \cdot d - \frac{d}{3} \cdot mg = 0$

$V = \frac{mg}{3}$ ins i 2

$S \cdot \frac{1}{\sqrt{2}} = mg - \frac{mg}{3}$ $\qquad 2 \cdot \sqrt{2}$

$S = \frac{\sqrt{2} \cdot 2\,mg}{3} = \frac{2 \cdot \sqrt{2}\,mg}{3}$ ins i 1

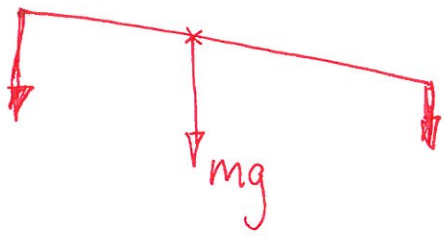
$1 \Rightarrow \frac{2 \cdot \sqrt{2} \cdot mg}{3} \cdot \frac{1}{\sqrt{2}} - H = 0$

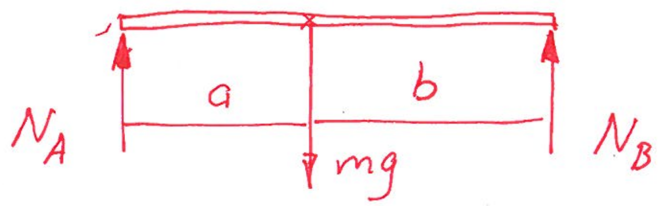
$H = \frac{2mg}{3}$

Svar: $V = \frac{mg}{3}$ $\quad H = \frac{2mg}{3}$ $\quad S = \frac{2 \cdot \sqrt{2}\,mg}{3}$

3,14

$b = 3a$




$\uparrow$: $N_A + N_B - mg = 0$ I

$\overset{\curvearrowleft}{A}$: $mg \cdot a - N_B(a+b) = 0$ II

$N_B = \underline{\underline{\frac{mga}{a+b}}}$ ins i I $\Rightarrow$

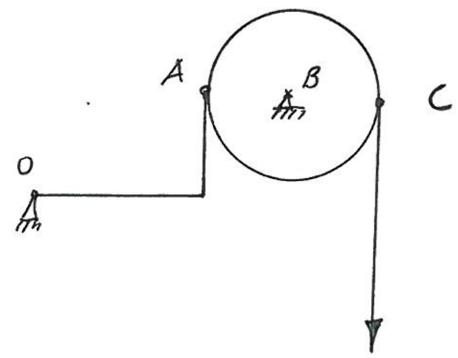
$N_A = mg - \frac{mga}{a+b} = \frac{mga + mgb}{a+b} - \frac{mga}{a+b} = \underline{\underline{\frac{mgb}{a+b}}}$

$b = 3a \Rightarrow$

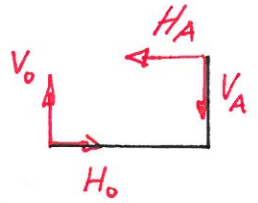
$N_A = \frac{mg \cdot 3a}{a+3a} = \frac{\cancel{a}\,3mg}{4\cancel{a}} = \underline{\underline{\frac{3mg}{4}}}$

$N_B = \frac{mga}{4a} = \underline{\underline{\frac{mg}{4}}}$

# 3,18




Lösning frilägg delarna och sätt ut krafter

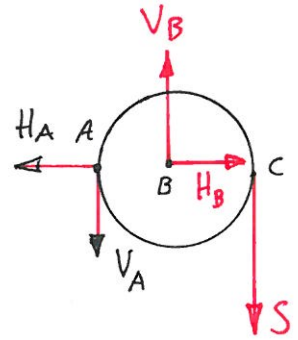



$\rightarrow$ : $H_0 - H_A = 0$

$\uparrow$ : $V_0 - V_A = 0$

$O\curvearrowright$ : $V_A \cdot 2r - H_A \cdot r = 0$

$$V_A = \frac{H_A}{2} \Rightarrow H_A = 2V_A$$




$\rightarrow$ : $-H_A + H_B = 0$ $\quad \Delta$

$\downarrow$ : $V_A - V_B + S = 0$ $\quad$ ✡

$A\curvearrowright$ : $S \cdot 2r - V_B \cdot r = 0$

$$V_B = \frac{2S \cdot r}{r} = 2S$$ ins i ✡

$V_A - 2S + S = 0$

$V_A = S$

$H_A = 2V_A = 2S$ ins i $\Delta \Rightarrow H_B = H_A = 2S$

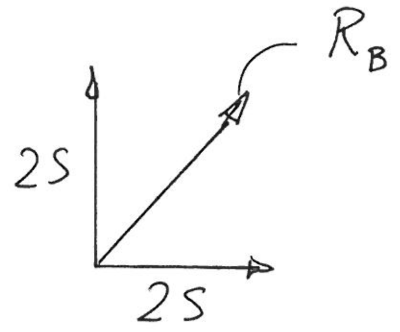
$\Longrightarrow$

Forts

3.18 forts.

$\Longrightarrow \quad H_B = 2S$

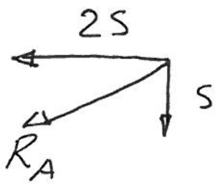
$V_B = 2S$




$$R_B = \sqrt{(2S)^2 + (2S)^2} = \sqrt{8S^2} = \sqrt{4} \cdot \sqrt{2} \cdot S$$

$$R_B = \underline{\underline{2 \cdot \sqrt{2}\, S}}$$

$\underline{R_A}$:

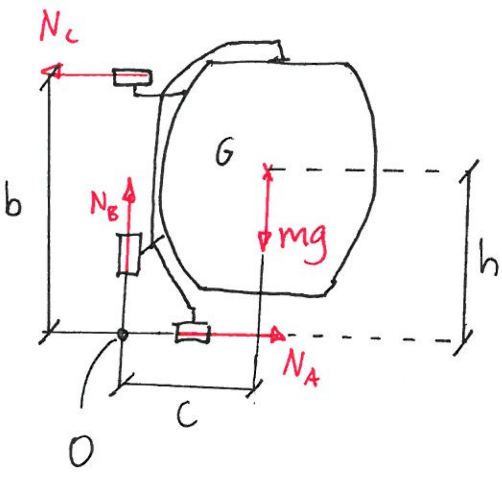



$$R_A = \sqrt{(2S)^2 + S^2} = \underline{\underline{\sqrt{5}\, S}}$$

Svar: $R_A = \sqrt{5} \cdot S$

$R_B = 2 \cdot \sqrt{2}\, S$

3,21




massan m

tyngd = mg

lösning frilägg sätt ut krafter.

$\rightarrow$ : $N_A - N_c = 0$

$\uparrow$ : $N_B - mg = 0$

$\curvearrowleft$ O : $mg \cdot c - N_c \cdot b = 0$

$N_B = mg \qquad N_A = N_c$

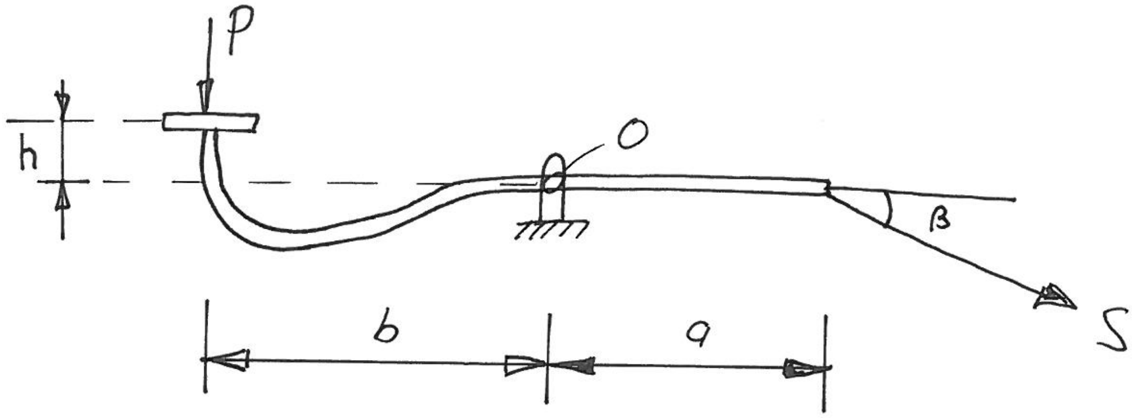
$$N_c = \frac{m \cdot g \cdot c}{b}$$

$$N_A = mg\frac{c}{b}$$

Eftersom det var två par hjul i varje punkt så är reaktionskrafterna hälften så stora

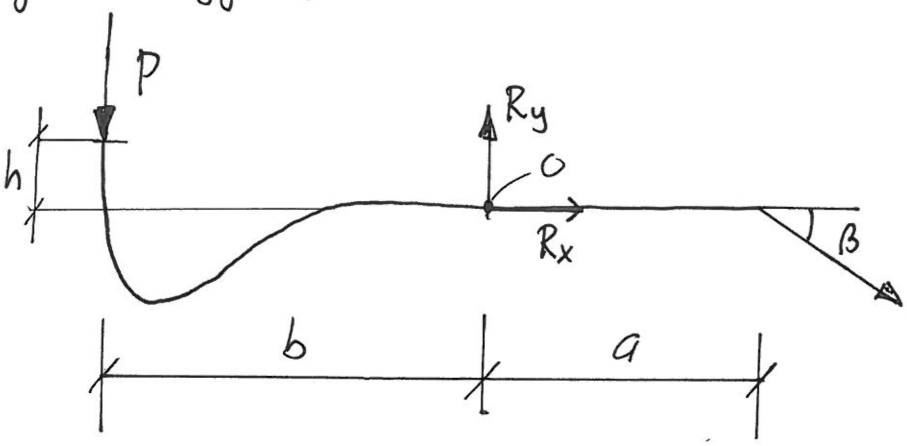
$$N_B = \frac{mg}{2}, \quad N_A = N_c = \frac{c}{2b}mg$$

3,33




Lösning: Frilägg systemet och sätt ut krafter

Sökt: S ?
$\left.\begin{matrix} R_x \\ R_y \end{matrix}\right\} R$




$\rightarrow$: $S \cdot \cos\beta + R_x = 0$ (1) $\Rightarrow R_x = -S \cdot \cos\beta$

$\uparrow$ : $R_y - P - S \cdot \sin\beta = 0$ (2) $\Rightarrow R_y = P + S \cdot \sin\beta$

$\circlearrowleft O$ : $S \cdot \sin\beta \cdot a - P \cdot b = 0$ (3)

$$S = \frac{P \cdot b}{a \cdot \sin\beta}$$ ins i (1) o (2)

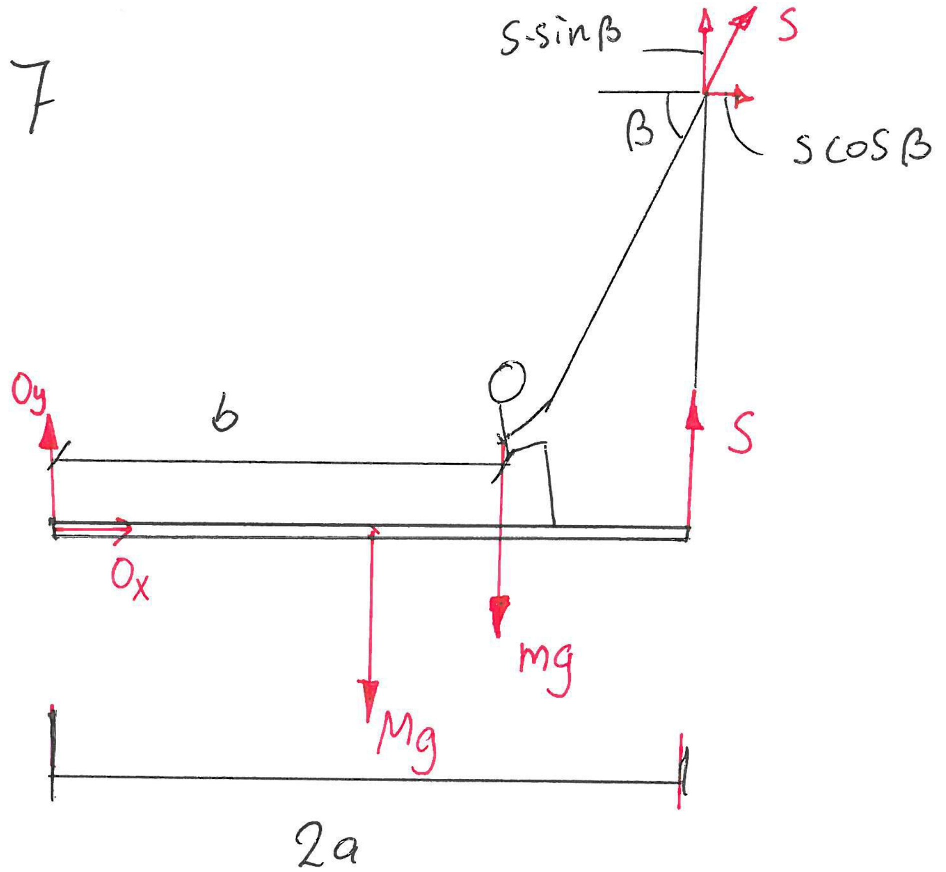
$$R_x = -\frac{P \cdot b}{a \cdot \sin\beta} \cdot \cos\beta = -\frac{b}{a \tan\beta} P$$

$$R_y = P + \frac{P \cdot b \cdot \cancel{\sin\beta}}{a \cdot \cancel{\sin\beta}} = P + \frac{Pb}{a} = \frac{P(a+b)}{a}$$

$$R = \sqrt{R_x^2 + R_y^2} = P\sqrt{\frac{(a+b)^2}{a^2} + \frac{b^2}{a^2 \tan^2\beta}} = \frac{P}{a}\sqrt{(a+b)^2 + \frac{b^2}{\tan^2\beta}}$$

Svar: $S = \frac{P \cdot b}{a \sin\beta}$ $R = \frac{P}{a}\sqrt{(a+b)^2 + \frac{b^2}{\tan^2\beta}}$

3.37
(XXX)




<u>Momentekvation</u> kring O ger oss direkt S:

$$\curvearrowleft O: S\cdot 2a + S\cdot \sin\beta\cdot 2a - S\cdot\cos\beta\cdot h - Mga - mgb = 0$$

$$S\cdot 2a + S\cdot\sin\beta\cdot 2a - S\cdot\cos\beta\cdot h = Mga + mgb$$

$$S = \frac{(Ma+mb)g}{(2a+2a\sin\beta) - \cos\beta\cdot h}$$

<u>Kraftjämvikt:</u> $\rightarrow$ $O_x + S\cos\beta = 0$

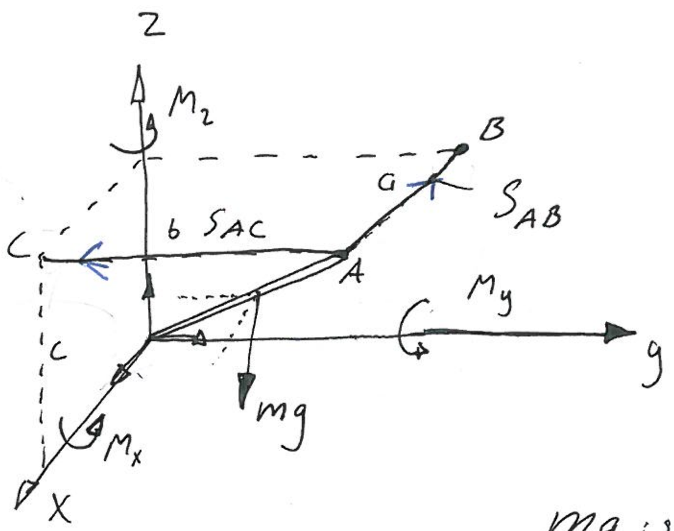
$$O_x = -S\cos\beta$$

$\uparrow$: $O_y + S + S\cdot\sin\beta - Mg - mg = 0$

$$O_y = (M+m)g - S - S\cdot\sin\beta = (M+m)g - S(1+\sin\beta)$$

Svar: $S = \dfrac{(Ma+mb)g}{2a(1+\sin\beta) - h\cdot\cos\beta}$ , $O_x = -S\cos\beta$
$O_y = (M+m)g - S(1+\sin\beta)$

$\boxed{3,59}$




mg:s läge $\left(\frac{a}{2}, \frac{b}{2}, \frac{c}{2}\right)$

$\boxed{\sum \bar{F} = 0}$ Bestäm $S_{AB}$ & $S_{AC}$

$$\left.\begin{aligned} \sum F_x &= 0 \Rightarrow & -S_{AB} + R_x &= 0 \\ \sum F_y &= 0 & -S_{AC} + R_y &= 0 \\ \sum F_z &= 0 & R_z - mg &= 0 \end{aligned}\right\} \begin{matrix} 1 \\ 2 \\ 3 \end{matrix}$$

$\sum \bar{M}_o = 0$

$$\sum M_{ox} = S_{AC} \cdot c - mg \cdot \frac{b}{2} = 0 \qquad 4$$

$$\sum M_{oy} = -S_{AB} \cdot c + mg \cdot \frac{a}{2} = 0 \qquad 5$$

$$\sum M_{oz} = -S_{AC} \cdot a + S_{AB} \cdot b = 0 \qquad 6$$

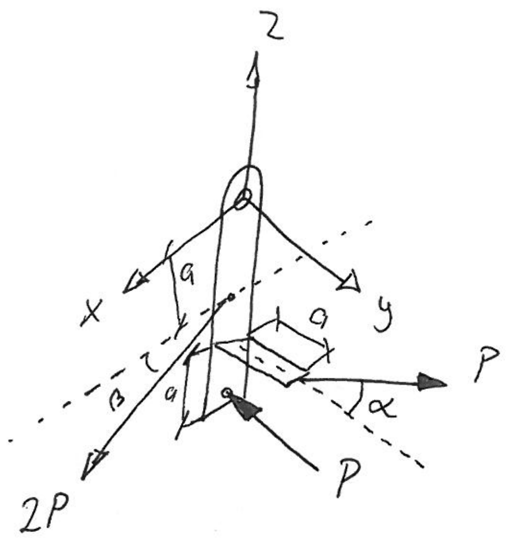
ekv 4 & 5 $\Rightarrow$ $S_{AC} = \dfrac{mgb}{2c}$

$$S_{AB} = \frac{mga}{2c}$$

Ekv 6 behövs ej men man kan kontrollera sin lösning mha att sätta in $S_{AC} = \dfrac{mgb}{2c} \Rightarrow -\dfrac{mgb \cdot a}{2c} + S_{AB} \cdot b = 0$

$S_{AB} \cancel{b} = \dfrac{mgba}{2cb} = \dfrac{mg\cancel{b}a}{2c \cdot \cancel{b}} = \dfrac{mga}{2c}$ Vilket stämmer.

3,64




↙ : x led $2P \cdot \cos\beta - P\sin\alpha$

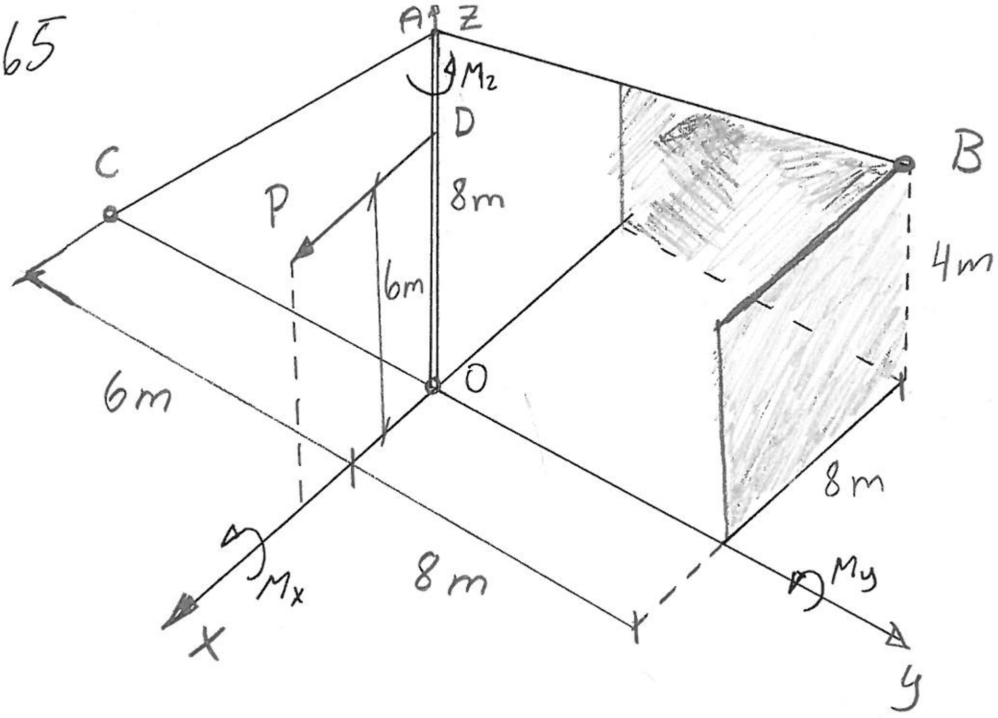
↘ : y: led: $P\cos\alpha - P$

↑ : z: led $-2P\sin\beta$

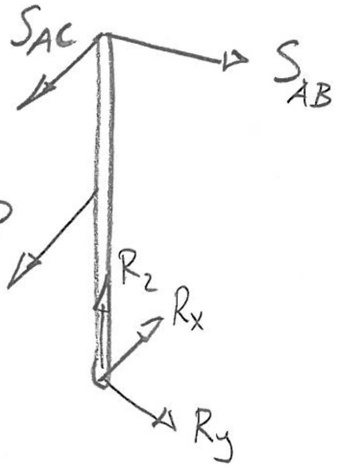
$$\bar{R} = \left(2P\cdot\cos\beta - P\sin\alpha\right)\hat{e}_x + \left(P\cos\alpha - P\right)\hat{e}_y - 2P\sin\beta\,\hat{e}_z$$

$$\bar{R} = \left(2P\cos\beta - P\sin\alpha,\ P\cos\alpha - P,\ -2P\sin\beta\right)$$

3,65




$P = 8\,kN$


Kraftekvationerna

$$\Sigma \bar{F} = \bar{0}$$

ger att 3 ekv 5 obekanta

Momentekvationerna

$$\Sigma \bar{M}_0 = \bar{0} \Rightarrow$$ 3 ekvationer och 2 obekanta alltså kan vi

Teckna $S_{AB}$ som en vektor

$$\bar{S}_{AB} = S_{AB} \cdot \hat{e}_{AB}$$

$$\hat{e}_{AB} = \frac{\overline{AB}}{|AB|} = \frac{(-8,8,4)-(0,0,8)}{|(-8,8,4)-(0,0,8)|} = \frac{(-8,8,-4)}{|(-8,8,-4)|}$$

$\Longrightarrow$

3,65 forts

$$= \frac{4(-2,2,-1)}{\sqrt{(-8)^2+8^2+(-4)^2}} = \frac{4(-2,2,-1)}{\sqrt{64+64+16}} = \frac{4(-2,2,-1)}{12}$$

$$= \frac{1}{3}(-2,2,-1)$$

$$\bar{S}_{AB} = \frac{S_{AB}}{3}(-2,2,-1)\ N$$

$$\hat{e}_{AC} = \frac{AC}{|AC|} = \frac{\bar{r}_C - \bar{r}_A}{|\bar{r}_C - \bar{r}_A|} = \frac{(0,-6,0)-(0,0,8)}{|(0,-6,0)-(0,0,8)|}$$

$$= \frac{(0,-6,-8)}{\sqrt{(-6)^2+(-8)^2}} = \frac{(0,-6,-8)}{\sqrt{36+64}} = \frac{2(0,-3,-4)}{10}$$

$$= \frac{(0,-3,-4)}{5}$$

$$\bar{S}_{AC} = \frac{S_{AC}}{5}(0,-3,-4)\ N$$

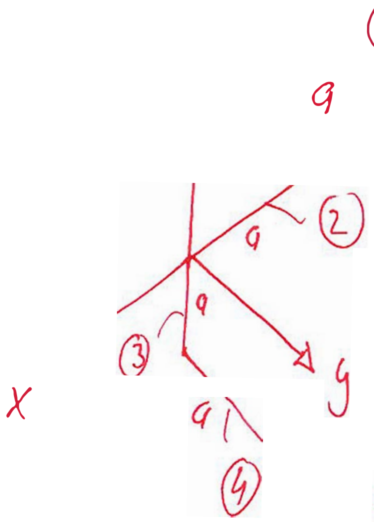
$$\Sigma \bar{M}_x = 0 \Rightarrow -\frac{S_{AB}\cdot 2}{3}\cdot 8 + \frac{S_{AC}\cdot 3}{5}\cdot 8 = 0 \Rightarrow S_{AB} = \frac{9 S_{AC}}{10}$$

$$\Sigma \bar{M}_y = 0 \Rightarrow P\cdot 6 - \frac{2 S_{AB}\cdot 8}{3} = 0$$

$$S_{AB} = \frac{6P\cdot 3}{2\cdot 8} = \frac{18P}{16} = \frac{9P}{8} \Rightarrow$$

$P = 8\,kN$ $$S_{AC} = \frac{10\cdot S_{AB}}{9} = \frac{10\cdot 9P}{8\cdot 9} = \frac{10P}{8}$$

$$\Rightarrow S_{AB} = 9\,kN \quad S_{AC} = 10\,kN$$




$$(l_1 + l_2 + l_3 + l_4) \cdot X_G$$

$$l_1 \cdot (-a) + l_2 \cdot -\frac{a}{2} + 0 + 0$$

$$4a X_G = \frac{a \cdot -a + a \cdot -\frac{a}{2}}{4a}$$

$$X_G = -\frac{3a}{8}$$

$$4a \cdot y_G = 0 + 0 + 0 + \frac{a}{2} \cdot a$$

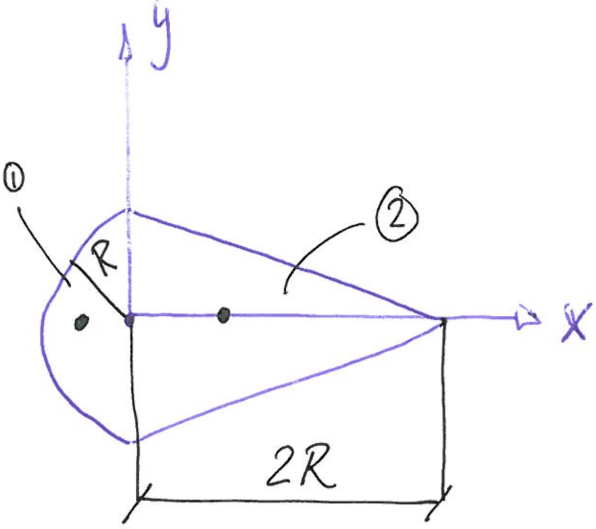
$$y_G = \frac{a}{8}$$

$$4a\, Z_G = \cancel{a \cdot \frac{a}{2}} + 0 + \cancel{a \cdot \left(-\frac{a}{2}\right)} + a(-a)$$

$$= -\frac{a}{4}$$

$$F_G = \left(-\frac{3a}{8} \,/\, \frac{a}{8} \,/\, \frac{-a}{4}\right)$$

4,2)




Pga symmetri ligger TP på x-axeln.

$$A_1 = \frac{\pi \cdot R^2}{2}$$

$$A_2 = \frac{2R \cdot 2R}{2}$$

$$(A_1 + A_2)\, x_G = A_2 \cdot x_2 - A_1 \cdot x_1$$

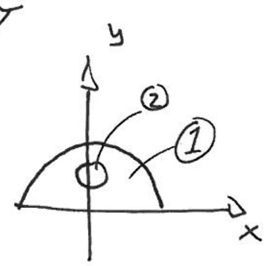
$$\left(\frac{\pi \cdot R^2}{2} + \frac{4R^2}{2}\right) x_G = \frac{4R^2 \cdot 2R}{2 \cdot 3} - \frac{\pi \cdot R^2}{2} \cdot \frac{4R}{3\cancel{\pi}}$$

$$x_G = \frac{\frac{8R}{3} - \frac{4R}{3}}{\pi + 4} = \frac{4R}{3(\pi + 4)}$$

4,3)

$$A_1 = \frac{\pi \cdot (6a)^2}{2} = \frac{36\pi a^2}{2} = 18a^2\pi$$

$$A_2 = \pi \cdot (2a)^2 = 4a^2\pi$$




$$(A_1 - A_2) \cdot y_G = A_1 \cdot y_1 - A_2 \cdot y_2$$

$$\pi a^2 (18 - 4) \cdot y_G = 18\pi \cdot a^2 \cdot \frac{4 \cdot 6a}{3\pi} - 4a^2 \cdot 3a \cdot \pi$$

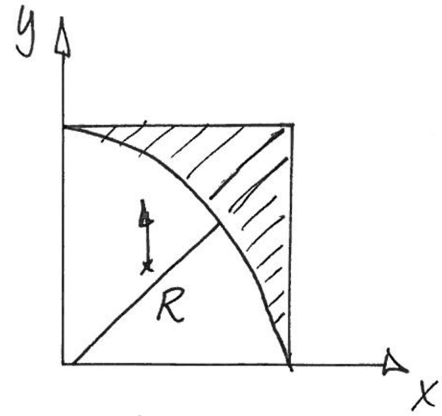
$$14\pi \cancel{a^2} y_G = \frac{\cancel{a^2}\, 18\cancel{\pi} \cdot 4 \cdot \cancel{6}^{2} a}{\cancel{3}\cancel{\pi}} - \cancel{a^2} \cdot 12a \cdot \pi$$

$$14\pi y_G = 144a - 12\pi a$$

$$y_G = \frac{12\pi a\left(\frac{12}{\pi} - 1\right)}{14\pi} = \frac{6a}{7}\left(\frac{12}{\pi} - 1\right)$$

Svar: $y_G = \frac{6a}{7}\left(\frac{12}{\pi} - 1\right) \approx \underline{\underline{2{,}4a}}$

4,4)

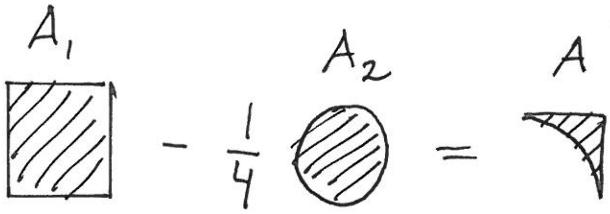



$A_1 = R^2$

$A_2 = \pi R^2$

$x_1 = R/2 = y_1$

$x_2 = \dfrac{4R}{3\pi} = y_2$




$$\left(R^2 - \frac{\pi R^2}{4}\right) x_G = R^2 \cdot \frac{R}{2} - \frac{\pi R^2}{4} \cdot \frac{4R}{3\pi}$$

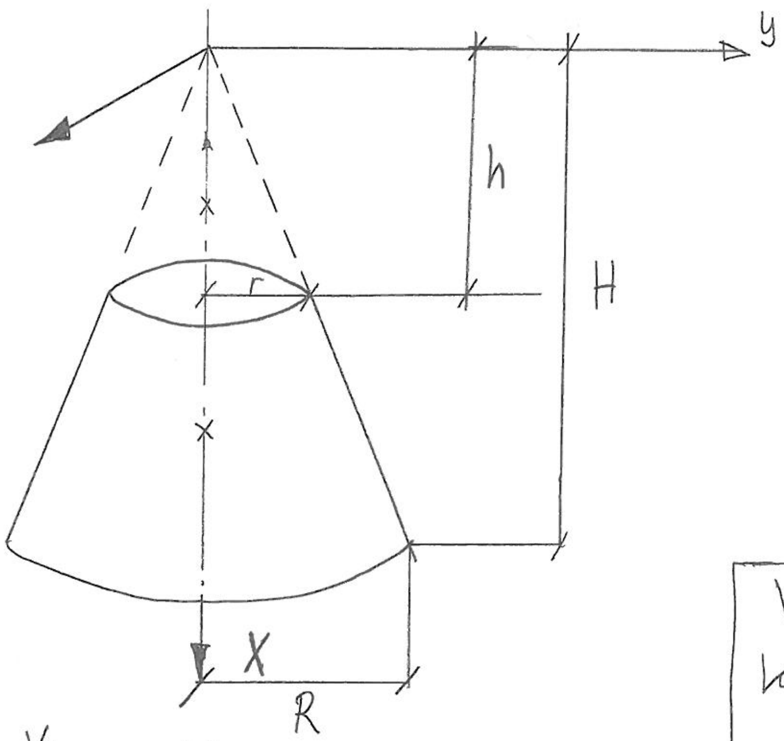
$$R^2\left(1 - \frac{\pi}{4}\right) x_G = \frac{R^3}{2} - \frac{\cancel{4}R^3}{\cancel{4}\cdot 3}$$

$$\left(1 - \frac{\pi}{4}\right) x_G = \frac{R}{2} - \frac{R}{3}$$

$$x_G = \frac{R}{6} \Big/ \left(1 - \frac{\pi}{4}\right) = \frac{R}{6\left(\frac{4-\pi}{4}\right)} = \frac{4R}{6(4-\pi)} = \frac{2R}{3(4-\pi)}$$

$$x_G = \frac{2R}{12 - 3\pi}$$ symmetri ger $$y_G = \frac{2R}{12 - 3\pi}$$

4,5




likformighet.

$$\left[\frac{h}{r} = \frac{H}{R}\right] \Rightarrow r = \frac{h \cdot R}{H}$$

$$\boxed{\begin{array}{l}\text{Volym rät cirkulär} \\ \text{kon} \quad \dfrac{\pi r^2 \cdot h}{3} \\ X_G = \dfrac{3h}{4}\end{array}}$$

$V_1$ $\quad V_2$ $\quad V$

$$V_1 = \frac{\pi \cdot R^2 \cdot H}{3} \qquad X_1 = \frac{3H}{4}$$

$$V_2 = \frac{\pi \cdot r^2 \cdot h}{3} = \frac{\pi \cdot \left(\frac{h \cdot R}{H}\right)^2 \cdot h}{3} \qquad X_2 = \frac{3h}{4}$$

$$(V_1 - V_2) X_G = V_1 \cdot X_1 - V_2 \cdot X_2$$

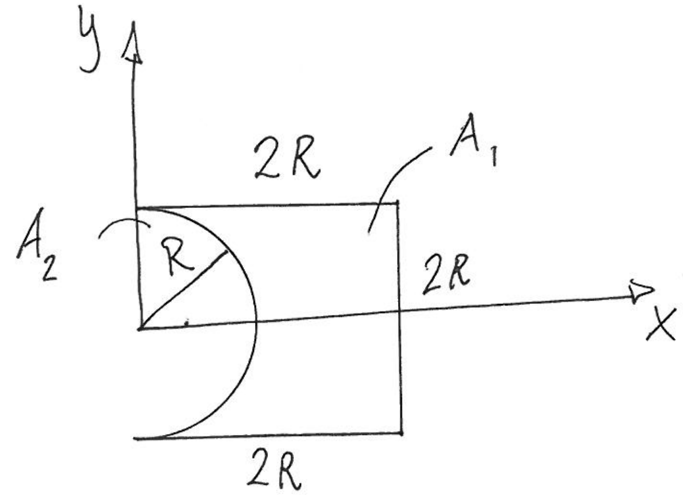
$$\left(\frac{\cancel{\pi} \cdot \cancel{R^2} \cdot H}{\cancel{3}} - \frac{\cancel{\pi} \cdot \frac{h^2 \cdot \cancel{R^2}}{H^2} \cdot h}{\cancel{3}}\right) X_G = \frac{\cancel{\pi} \cdot \cancel{R^2} \cdot H \cdot 3H}{\cancel{3} \cdot 4} - \frac{\cancel{\pi} \cdot h^3 \cdot \cancel{R^2} \cdot 3h}{\cancel{3} \cdot H^2 \cdot 4}$$

$$\left(H - \frac{h^3}{H^2}\right) X_G = \frac{3H^2}{4} - \frac{3h^4}{H^2 \cdot 4}$$

$$(H^3 - h^3) X_G = \frac{3H^4}{4} - \frac{3h^4}{4}$$

$$X_G = \frac{3(H^4 - h^4)}{4(H^3 - h^3)}$$

4.8)




På grund av symmetri så sammanfaller mass-centrum längs x-axeln.

□ $A_1$    D $A_2$      $A = A_1 - A_2$

$$A_1 = 2R \cdot 2R = 4R^2$$

$$A_2 = \frac{\pi \cdot R^2}{2}$$

$$(A_1 - A_2)\, X_G = A_1 \cdot X_1 - A_2 \cdot X_2$$

$$\left(4R^2 - \frac{\pi R^2}{2}\right) X_G = 4R^2 \cdot R - \frac{\pi R^2}{2} \cdot \frac{4R}{3\pi}$$

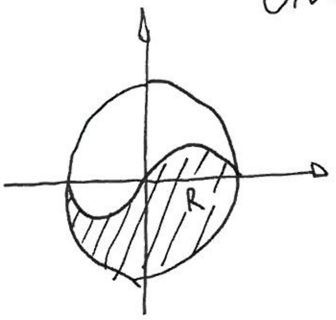
$$R^2\left(4 - \frac{\pi}{2}\right) X_G = R^2\left(4R - \frac{4R}{2 \cdot 3}\right)$$

$$\left(4 - \frac{\pi}{2}\right) X_G = \frac{8R}{2} - \frac{4R}{2 \cdot 3}$$

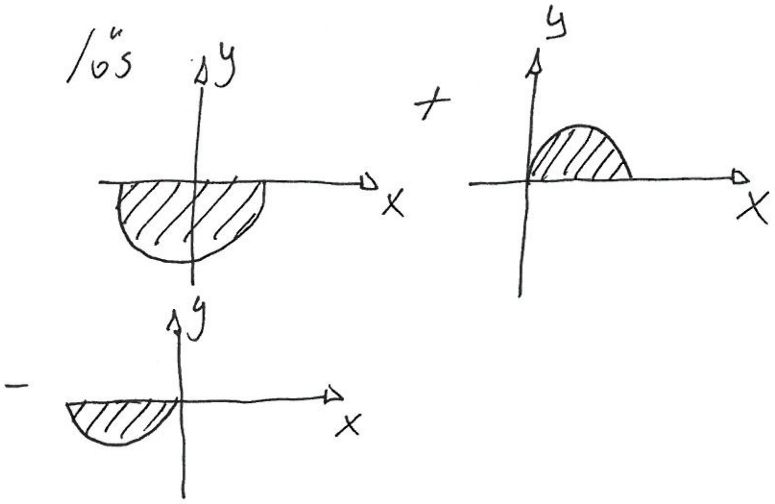
$$X_G = \frac{\frac{24R}{6} - \frac{4R}{6}}{\left(4 - \frac{\pi}{2}\right)} = \underline{\underline{\frac{20R}{6\left(4 - \frac{\pi}{2}\right)}}}$$

4,10]

Givet



Lös



Lösning:
$$X_T = \frac{\sum A_i \cdot x_i}{\sum A_i} = \frac{\frac{\pi R^2}{2}\cdot 0 + \frac{\pi\left(\frac{R}{2}\right)^2}{2}\cdot\frac{R}{2} - \frac{\pi\left(\frac{R}{2}\right)^2}{2}\left(-\frac{R}{2}\right)}{\frac{\pi R^2}{2} + \frac{\pi R^2}{8} - \frac{\pi R^2}{8}}$$

$$= \frac{R}{4}$$

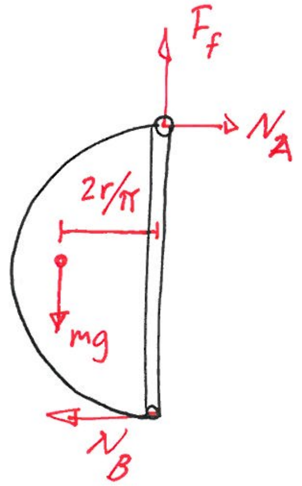
$$Y_T = \frac{\sum A_i \cdot y_i}{\sum A_i} = \frac{\frac{\pi R^2}{2}\left(-\frac{4R}{3\pi}\right) + \frac{\pi R^2}{8}\cdot\frac{4\frac{R}{2}}{3\pi} - \frac{\pi R^2}{8}\left(\frac{-4\frac{R}{2}}{3\pi}\right)}{\frac{\pi R^2}{2}}$$

$$Y_T = \frac{\frac{\cancel{2}\cdot\cancel{\pi}\cancel{R^2}}{\cancel{2}}\left(-\frac{4R}{3\cancel{\pi}}\right) + \frac{\cancel{2}\cancel{\pi}\cancel{R^2}}{\cancel{8}\,4}\cdot\frac{2R}{3\cancel{\pi}} + \frac{\cancel{2}\cancel{\pi}\cancel{R^2}}{\cancel{8}\,4}\cdot\left(\frac{2R}{3\cancel{\pi}}\right)}{\pi\cdot\cancel{R^2}}$$

$$Y_T = \frac{-\frac{4R}{3} + \frac{2R}{12} + \frac{2R}{12}}{\pi} = \frac{R\left(\frac{2}{6} - \frac{4}{3}\right)}{\pi} = \frac{R\left(\frac{1}{3} - \frac{4}{3}\right)}{\pi}$$

$$Y_T = -\frac{R}{\pi}$$

5,1)




$\rightarrow \quad N_A - N_B = 0 \qquad (1)$

$\uparrow \quad F_F - mg = 0 \qquad (2)$

$B: \quad mg \cdot \frac{2\pi}{\pi} - N_A \cdot 2r = 0 \qquad (3)$

$$F_F = mg$$

$$N_A = \frac{mg \cdot 2r}{\pi \cdot 2r} = \frac{mg}{\pi}$$

Friktionsvillkoret

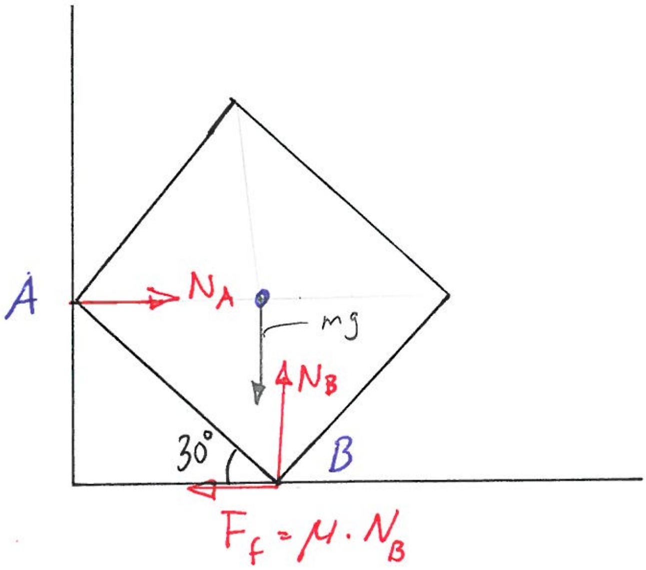
$F_f \leq \mu \cdot N_A$

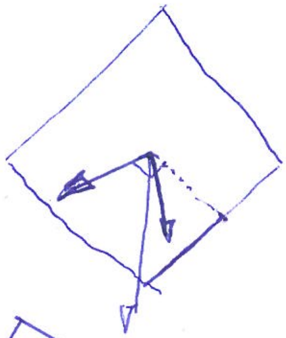
$F_f \leq \frac{\mu mg}{\pi} \Rightarrow mg \leq \frac{\mu mg}{\pi}$

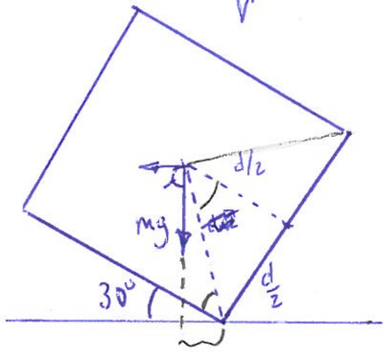
$$\underline{\underline{\mu \geq \pi}}$$

5,2]

Homogen kub






$$\sqrt{\left(\frac{d}{2}\right)^2+\left(\frac{d}{2}\right)^2}$$

$$\sqrt{\frac{2d^2}{4}} = \frac{d}{\sqrt{2}}$$

$\rightarrow$ : $N_A - F_F = 0$ (1)

$\uparrow$ : $N_B - mg = 0$ (2)

$\curvearrowleft B$ $\underbrace{mg \cdot \frac{d}{\sqrt{2}} \cdot \cos 75}_{\text{momentarm till mg}} - N_A \cdot \frac{d}{2} = 0$ (3)

$F_F = N_A \Rightarrow$

$$\frac{mg}{\sqrt{2}} \cdot d \cdot \cos 75 - F_F \cdot \frac{d}{2} = 0$$

$$F_F = \frac{2mg \cancel{d} \cdot \cos 75}{\cancel{d} \cdot \sqrt{2}} = \frac{2mg \cos 75}{\sqrt{2}}$$

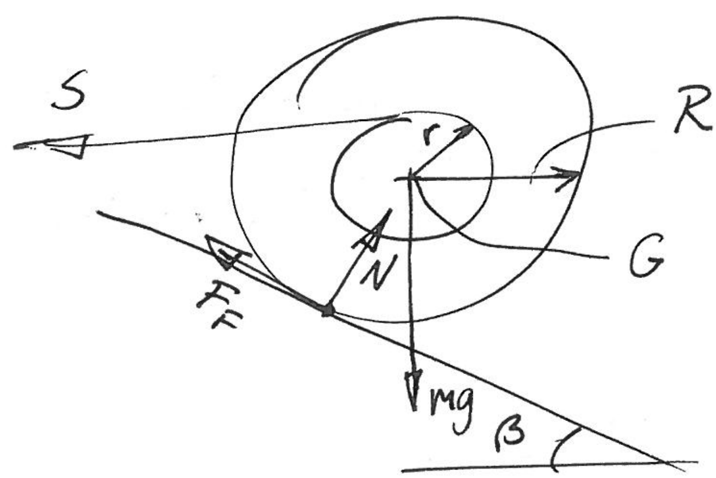
$N_B = mg$

Friktionsvillkoret $F_F = \mu \cdot N_B \Rightarrow$

$$\frac{2 \cancel{mg} \cos 75^\circ}{\sqrt{2}} = \mu \cancel{mg} \Rightarrow \mu = \sqrt{2} \cos 75^\circ$$

$\mu \approx 0,36$ = OK.

5,3




massan $m$

yttre radie $R$

inner radie $r$

$S = ?$

$\mu_s$

$F_{max} = \mu_s \cdot N$

$$\rightarrow: \; -F_F \cdot \cos\beta + N \cdot \sin\beta - S = 0 \qquad (1)$$

$$\uparrow: \; N \cdot \cos\beta + F_F \cdot \sin\beta - mg = 0 \qquad (2)$$

$$\circlearrowright G: \; F_F \cdot (R) - S \cdot r = 0 \qquad (3)$$

$$3 \Rightarrow S = \frac{R \cdot F_F}{r} \quad \text{ins i } 1.$$

$$\Rightarrow -F_F \cos\beta + N \cdot \sin\beta - \frac{R F_F}{r} = 0$$

$$N = \frac{(R + r\cos\beta) F_F}{r \sin\beta}$$

ins i ekv (2) ger

$$\frac{\cos\beta}{\sin\beta}\left(\frac{R}{r} + \cos\beta\right) F_F + F_F \cdot \sin\beta = mg$$

$$F_F = \frac{mgr\sin\beta}{r + R\cos\beta}$$

$$N = \frac{R + r\cos\beta}{r + R\cos\beta} mg \Rightarrow S = \frac{mgR\sin\beta}{r + R\cos\beta}$$

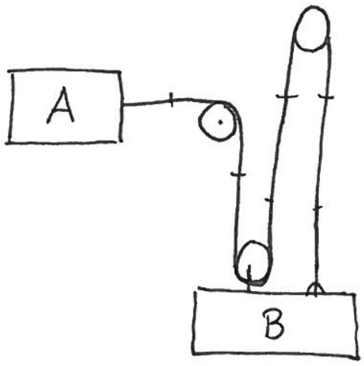
$\Longrightarrow$

5,3 forts.) $F_F \leq \mu \cdot N$

$$\frac{mgr\sin\beta}{r + R\cos\beta} \leq \mu \frac{R + r\cos\beta}{r + R\cos\beta} mg$$

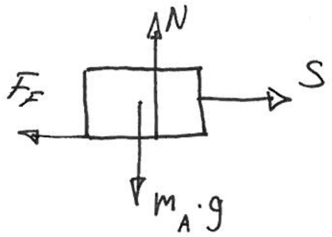
$$\Rightarrow \mu \geq \frac{r \cdot \sin\beta}{R + r\cos\beta}$$

5,4]

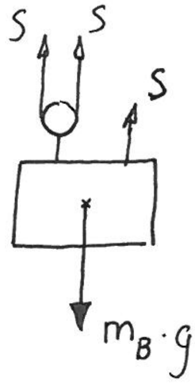



Frilägg A o B

A




B




Jämvikt B:

$\uparrow: 3S - m_B \cdot g = 0$

$S = \dfrac{m_B \cdot g}{3}$

Jämvikt för A:

$\rightarrow \quad S - F_F = 0 \quad *$

$\uparrow: \quad N - m_A \cdot g = 0$

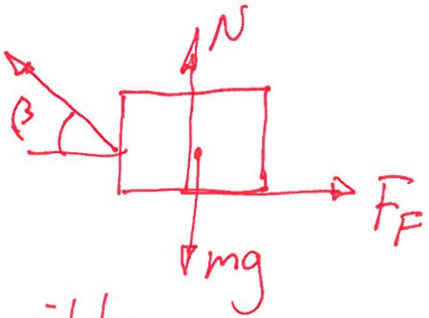
$\Rightarrow \quad F_F = \dfrac{m_A \cdot g}{3}$ och $N = m_A \cdot g$

Friktionsvillkoret $F_F \leq \mu \cdot N$

Vid gränsfall för glidning:

$* \quad \dfrac{m_B\, g}{3} \leq \mu \cdot m_A \cdot g \Rightarrow \quad \underline{\underline{\mu \geq \dfrac{m_B}{3 m_A}}}$

5.5]




jämvikt:

$\rightarrow$ : $-S \cdot \cos\beta + F_F = 0$ (1)

$\uparrow$ : $S \cdot \sin\beta + N - mg = 0$ (2)

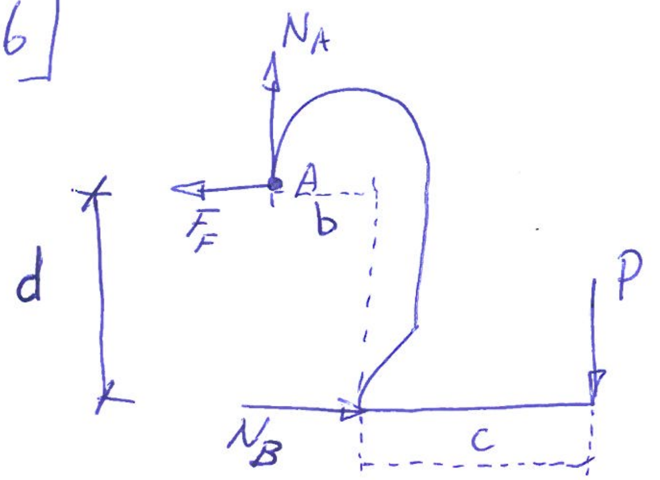
$F_F = \mu \cdot N$ (3)

$$\left.\begin{array}{l} -S\cos\beta + \mu \cdot N = 0 \\ \mu \cdot S\sin\beta + \mu N - \mu mg = 0 \end{array}\right\}$$

$$\Rightarrow \mu \cdot S \sin\beta + S \cdot \cos\beta - \mu mg = 0$$

$$S = \frac{\mu mg}{\mu \cdot \sin\beta + \cos\beta}$$

5.6




$\rightarrow$: $N_B - F_F = 0$ (1)

$\uparrow$: $N_A - P = 0$ (2)

$\curvearrowleft A$: $N_B \cdot d - P(b+c) = 0$ (3)

$N_A = P$

$N_B = F_F$ ins i (3) $\Rightarrow$

$F_F \cdot d - P \cdot (b+c) = 0$

$$F_F = \frac{P(b+c)}{d}$$

$F_F = \mu \cdot N_A$

$$\frac{P(b+c)}{d} = \mu \cdot P$$

$$\mu = \frac{b+c}{d}$$

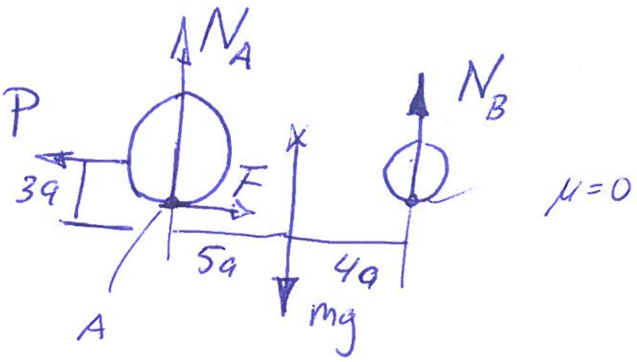
P = obegränsad

tills materialet går sönder.

5,8] $mg = 40\,kN$

$P = 4\,kN$

$\mu = 0{,}6$




$\uparrow$ $N_A + N_B - mg = 0$

$\rightarrow$: $F - P = 0$

$\overset{\curvearrowright}{A}$: $mg \cdot 5a - N_B \cdot 9a - P \cdot 3a = 0$

$$N_B = \frac{mg \cdot 5\cancel{a} + P \cdot 3\cancel{a}}{9\cancel{a}} = \frac{mg \cdot 5 - 3P}{9}$$

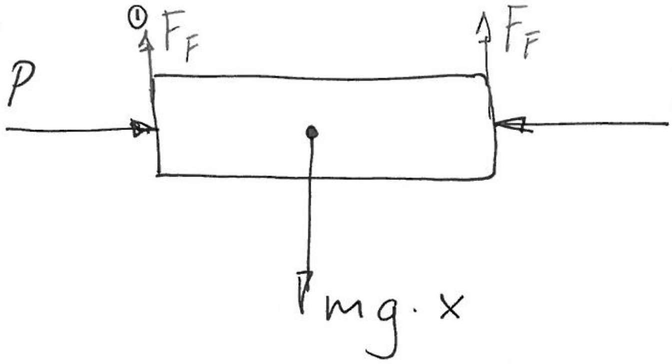
$$N_B = \frac{5 \cdot 40 - 3 \cdot 4}{9} = \underline{\underline{20{,}9\,kN}}$$

$$N_A = mg - N_B = 40 - 20{,}9 = \underline{\underline{19{,}1\,kN}}$$

$$F_F = \mu \cdot N_A = 0{,}6 \cdot 19{,}1 = 11{,}46\,kN$$

dvs friktionskraften är så pass stor att traktorn inte kommer att slira och kan således dra 4 kN utan problem.

5,9




$m = 1\ kg$

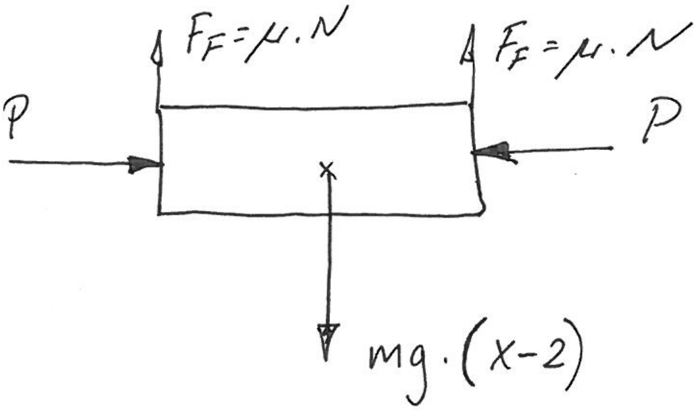
Fall 1
hand bok

$$mg \cdot x = 2 \cdot 0,5 \cdot 110$$

$$x = 11,2 = 11 \text{ böcker}$$

Fall2

Friktion bok↔bok

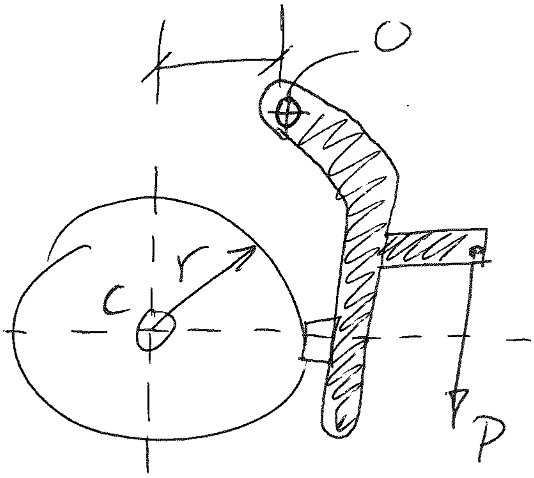



$$mg \cdot (x-2) \cdot = 0,4 \cdot 2 \cdot 110$$

$$x = \frac{0,4 \cdot 2 \cdot 110 + 2mg}{mg}$$

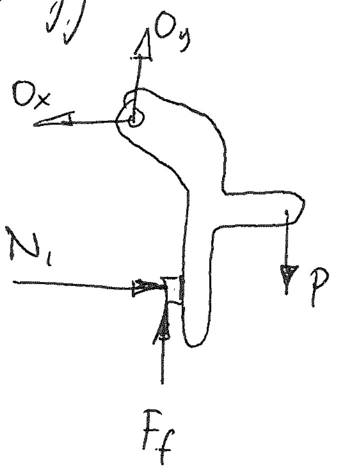
$$x = 10,96 \Rightarrow \underline{\underline{10 \text{ st böcker}}}$$

5,11)




Bestäm P om friktionstalet är $\mu$

Frilägg armen OA




$$\Sigma \bar{F} = \bar{0}$$
$$\Sigma \bar{M} = \bar{0}$$

$$\curvearrowleft O: \; F_F \cdot (r-a) + N_1 \cdot (b) - P \cdot (c-a) = 0$$

$$F_F = \mu \cdot N_1 \;\Rightarrow\; N_1 = \frac{F_F}{\mu}$$

$$F_F(r-a) - \frac{F_F \cdot b}{\mu} - P(c-a) = 0$$

$$P = \left(\frac{(r-a) - \frac{b}{\mu}}{(c-a)}\right) F_F = \left(\frac{\mu(r-a) - b}{\mu(c-a)}\right) F_F$$

Eftersom man inte vet om hjulet roterar medurs eller moturs måste man föra in $\pm$ i ekvationen

P blir då

$$P = \frac{b \pm \mu \cdot (r-a)}{\mu(c-a)} F_F$$

(Plus då hjulet roterar moturs)